T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE 2010

Paper I

PART I

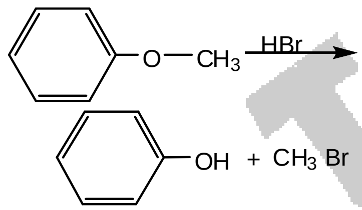
1	2	3	4	5	6	7	8
D	**A**	**B**	**B**	**C**	**C**	**D**	**A**

9	10	11	12	13
B, D	**B, C**	**A, B**	**C, D**	**A, C**

14	15	16	17	18
A	**D**	**C**	**B**	**C**

19	20	21	22	23	24	25	26	27	28
3	**2**	**8**	**3**	**4**	**3**	**4**	**1**	**0**	**5**

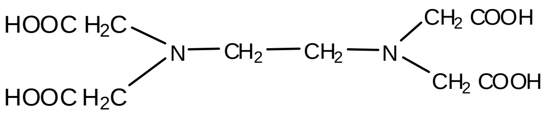
Section I

1. 

2. k varies exponentially with T.

3. Standard enthalpy of formation of elements in their standard state is taken as zero.

4. Ligand NO_2^- and counter ion Cl^- are mutually exchanged

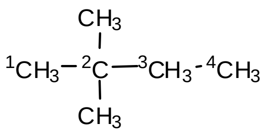
5. EDTA is


6. Bond energy of C–C is 83Kcalmol^{-1}

7. $CH_3 - CH_2 - C \equiv CH$ $\xrightarrow{NaNH_2}$ $CH_3 - CH_2 - C \equiv CNa$ $\xrightarrow{BrCH_2-CH_2-CH_2-CH_3}$
$CH_3 - CH_2 - C \equiv C - CH_2 - CH_2 - CH_2 - CH_3$
3-octyne

8. Ring (a) is pyranose with α- glycosidic link.

Section II

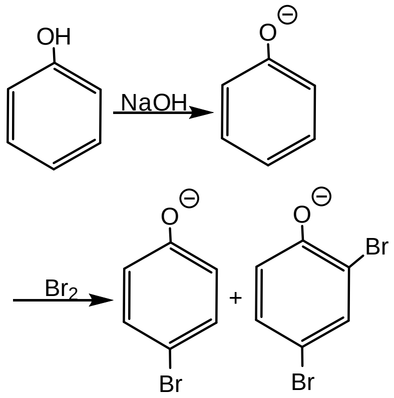
9. 

2, 2-dimethyl butane

10. $Ca(OH)_2$ and Na_2CO_3 can be used for softening temporary hard water.

11. Molar conductivity and electromotive force are intensive properties.

12. HNO_3 and CH_3COONa – HNO_3 on mixing with excess CH_3COONa also results in a mixture of CH_3COOH and CH_3COONa.

13. 

etc.

Section III

14. Cu FeS_2 is converted to Cu_2S, FeS and SO_2. FeS is further oxidized to FeO.

15. FeO forms $FeSiO_3$ with SiO_2

16. Cu(I) is reduced to Cu(0) by S^{2-} which gets oxidized to SO_2

17. $E_{cell} > 0$ and $\Delta G < 0$
It is given that E_{cell} = 70 mV
For a concentration cell $E^\circ_{cell} = 0$
$\therefore \Delta G^\circ = 0$

18. $70 = 0.06 \log \frac{1}{.05}$

$\therefore 0.06 \log \left(\frac{1}{0.05}\right)^2 = 2 \times 70 = 140 mV$

Section IV

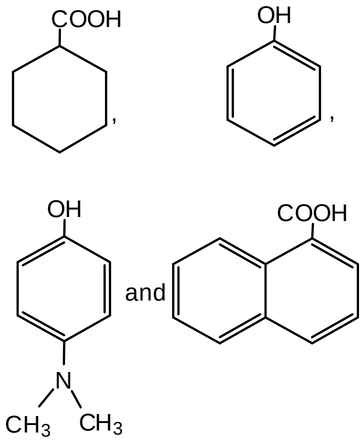
19. It is $Be_3Al_2Si_6O_{18}$

20. $-NH_2$ and $-COO^{\ominus}$ groups are basic groups.

21. BrF_5 is square pyramidal the F–Br–F angle is 90°. 4 bond angle between Br–F bonds on the plane and 4 with the axial Br–F bond.

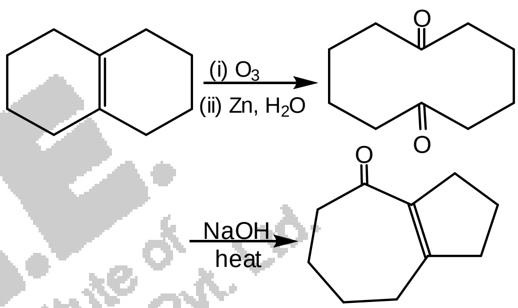
22. KCN, K_2CO_3 and LiCN are the salts of strong bases with weak acids.

23. The compounds soluble in aqueous NaOH are



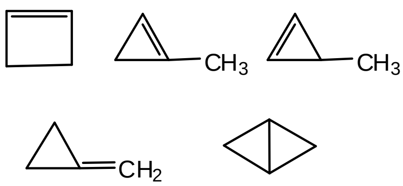
24. 25.2 and 25.0 has only one digit after decimal point.

25. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{142}_{54}Xe + ^{90}_{38}Sr + 4^{1}_{0}n$

26. 

Since the dicarbonyl compound formed is symmetrical, if can form only one intramolecular aldol condensation product.

27. $\frac{dc}{dt} = $ a constant (5)

28. 

PART II

29	30	31	32	33	34	35	36
A	**B**	**B**	**C**	**D**	**D**	**B**	**A**

37	38	39	40	41
A, C, D	**B or D**	**A**	**A**	**B**

42	43	44	45	46
D	**C**	**D**	**B**	**A**

47	48	49	50	51	52	53	54	55	56
1	**3**	**4**	**6**	**2**	**1**	**9**	**5**	**7**	**2**

Section I

29. Non–homogenous linear systems of equations, three can be only 3 options.
    (a) unique solution
    (b) no solution
    (c) infinity may solution
    Hence these cannot be a stem with exactly two distinct solutions

30. Limit = $\lim_{x \to 0} \dfrac{x\ln(1+x)}{(x^4+4) \times 3x^2}$

$\lim_{x \to 0} \dfrac{\ln(1+x)}{3x(x^4+4)} \left(\dfrac{0}{0}\right)$

$\lim_{x \to 0} \dfrac{1}{3(1+x)(x^4+4)} = \dfrac{1}{12}$

31. $\alpha + \beta = -p$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

$q = -p^3 + 3 - p\alpha\beta$

$3p\alpha\beta = q + p^3$

$\alpha\beta = \dfrac{q+p^3}{3p}$

sum of the roots = $\dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{p^2 - \dfrac{2(q+p^3)}{3p}}{\left[\dfrac{(q+p^3)}{3p}\right]}$

$= \dfrac{3p^3 - 2(q+p^3)}{3p} \times \dfrac{3p}{(q+p^3)}$

$= \dfrac{p^3-2q}{(q+p^3)}$

Product of the roots = 1

Equation is

$x^2 - \dfrac{(p^3-2q)}{(q+p^3)}x + 1 = 0$

$(q + p^3)x^3 - (p^3 - 2q)x + (q + p^3) = 0$

32. Ax + By + Cz = 0 — (1)

2A + 3B + 4C = 0

8A – B – 10C = 0

2	3	4
8	–1	–10

–30 + 4,	32 + 20,	–26
–26,	52,	–26
–1,	2,	–1

–x + 2y – z = 0

x – 2y + z = 0

33. $\dfrac{\text{SinA}}{\text{SinC}}$ 2 SinC CosC

2SinA CosC + $\dfrac{\text{SinC}}{\text{SinA}}$ 2SinA CosA

2SinA CosC + 2 CosA SinC

(2Sin(A + C)

2Sin 120 = $2 \times \dfrac{\sqrt{3}}{2} = \sqrt{3}$

34. $f(x) = e^{x^2} + e^{-x^2}$

$g(x) = xe^{x^2} + e^{-x^2}$

$h(x) = x^2e^{x^2} + e^{-x^2}$

$f' = 2xe^{x^2} - 2xe^{-x^2}$

$= 2xe^{x^2} - \dfrac{2x}{e^{x^2}}$

$$= \frac{\left(2xe^{2x^2} - 2x\right)}{ex^2}$$

$$g' = xe^{x^2} \times 2x + e^{x^2} - 2xe^{-x^2}$$

$$= e^{x^2}(2x^2 + 1) - \frac{2x}{e^{x^2}} = \frac{\left[(2x^2+1)e^{2x^2} - 2x\right]}{e^{x^2}}$$

$$h' = x^2e^{x^2} \times 2x + e^{x^2} \times 2x - 2xe^{-x^2}$$

$$= 2x^3e^{x^2} + 2xe^{x^2} - 2xe^{-x^2}$$

$$= \frac{e^{2x^2}(2x^3 - 2x) - 2x}{e^{x^2}}$$

$$f' = 0 \rightarrow x = 0 \text{ or } e^{2x^2} = 1$$

$$2x^2 =$$

Max f(x) = $e + \frac{1}{e}$

Max g(x) = $e + \frac{1}{e}$

Max h(x) = $e + \frac{1}{e}$

a = b = c

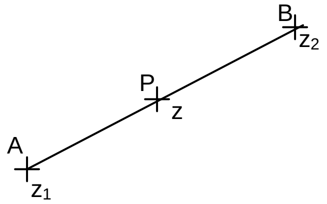
35. $1, 2, 3 \rightarrow 6$
$2, 3, 4 \rightarrow 6$
$3, 4, 5 \rightarrow 6$
$4, 5, 6 \rightarrow 6$

$$\frac{24}{6 \times 6 \times 6} = \frac{4}{6 \times 6} = \frac{1}{9}$$

36. PQ = 6i + j; QR = –i + 3j;
RS = –6j – j; SP = i – 3j
PQ || RS and QR || SP.
Also mid point of PR & QS in the same.
But |SP| ≠ |PQ| and $\overline{SP} \cdot \overline{PQ} \neq 0$
∴ It is a parallelogram but neither a rhombus nor a rectangle

Section II

37.



Since z = (1 – t) z_1 + t_{z2} for 0 < t < 1, z lies on the line segment joining z_1 and z_2

$|z - z_1| + |z - z_2|$ = PA + PB
= AB = $|z_1 - z_2|$

Arg $(z - z_1)$ angle made by AP with x – axis

Arg $(z - z_2)$ angle made by AP with x – axis

Arg $(z_2 - z_1)$ angle made by AP with x – axis

Arg $(z - z_1)$ = Arg $(z_2 - z_1)$

$$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix}$$

= $z - z_1 = |z - z_1|$ arg $(z - z_1)$

$z_2 - z_1 = |z_2 - z_1|$ arg $|z_2 - z_1|$

$$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix}$$

$$= \begin{vmatrix} |z - z_1| \arg(z - z_1) & -|z - z_1| \arg(z - z_1) \\ |z_2 - z_1| \arg(z_2 - z_1) & -|z_2 - z_1| \arg(z_2 - z_1) \end{vmatrix} = 0$$

38. $(t_1^2, 2t_1)$
$(t_2^2, 2t_2)$
$(x - t_1^2)(x - t_2^2) + (y - 2t_1)(y - 2t_2) = 0$
y = 0
$(x - t_1^2)(x - t_2^2) + 4t_1t_2 = 0$
$x^2 - (t_1^2 + t_2^2)x + t_1^2 t_2^2 + 4t_1t_2 = 0$
$(t_1^2 + t_2^2)^2 = 4(t_1^2t_2^2 + 4t_1t_2)$
$(t_1^2 - t_2^2)^2 = 16t_1t_2$ — (1)

$$\left[\frac{t_1^2 + t_2^2}{2}, (t_1 + t_2)\right]$$

$(t_1^2, 2t_1)$

$$r^2 = \left[t_1^2 - \frac{t_1^2 + t_2^2}{2}\right]^2 + (t_1 + t_2)^2$$

$$= \frac{(t_1^2 - t_2^2)^2}{4} + (t_1 + t_2)^2$$

$$\frac{(t_1^2 - t_2^2)^2}{4} = r^2 - (t_1 - t_2)^2 \text{ — (2)}$$

$4[r^2 - (t_1 - t_2)^2] = 16t_1t_2$

$r^2 = (t_1 - t_2)^2 + 4t_1t_2 \quad = (t_1 + t_2)^2$

$t_1 + t_2 = r$

$$\frac{2}{t_1 + t_2} = \frac{2}{r} \text{ or } \frac{-2}{r}$$

39. $f(x) = \ell n + \int_0^x \sqrt{1 + \sin t}\, dt$

$$= \ell nx + \int_0^x \left(\sin\frac{t}{2} + \cos\frac{t}{2}\right) dt$$

$$= \ell n\, x + \left(-2\cos\frac{t}{2} + 2\sin\frac{t}{2}\right)_0^x$$

$$= \ell n\, x - 2\cos\frac{x}{2} + 2\sin\frac{x}{2} - (-2)$$

$$= \ell nx - 2\cos\frac{x}{2} + 2\sin\frac{x}{2} + 2$$

$$f'(x) = \frac{1}{x} + \sin\frac{x}{2} + \cos\frac{x}{2}$$

$$f''(x) = \frac{-1}{x^2} + \frac{1}{2}\cos\frac{x}{2} - \frac{1}{2}\sin\frac{x}{2}$$

f"(x) exists for all x ∈ (0, ∞)

40. $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$

$= \int_0^1 \frac{x^4(1+x^2-2x)^2}{1+x^2}dx$

$= \int_0^1 \left[x^4(1+x^2)+\frac{4x^6}{1+x^2}-4x^5\right]dx$

$= \frac{12}{35}+4\int_0^1 \frac{x^6}{1+x^2}dx-\frac{2}{3}$

$= \frac{12}{35}+4I-\frac{2}{3}$ — (1)

$I = \int_0^1 \frac{x^6}{1+x^2}dx$

$= \int_0^{\pi/4} \tan^6\theta d\theta = T_6$ (say)

$T_n = \int_0^{\pi/4} \tan^n \theta d\theta$

$= \int_0^{\pi/4} \tan^{n-2}\theta(\sec^2\theta-1)d\theta$

$= \left[\frac{\tan^{n-1}\theta}{(n-1)}\right]_0^{\pi/4} - T_{n-2}$

$T_n + T_{n-2} = \frac{1}{n-1}$

$T_6 + T_4 = \frac{1}{5}$

$T_4 + T_2 = \frac{1}{3}$

$T_6 = \frac{1}{5} - T_4$

$= \frac{1}{5} - \left(\frac{1}{3} - T_2\right)$

$= \frac{-2}{15} + \int_0^{\pi/4}(\sec^2\theta - 1)d\theta$

$= \frac{-2}{15} + 1 - \frac{\pi}{4} = \frac{13}{15} - \frac{\pi}{4}$

Hence, on substituting in (1)

$\int_0^1 = \frac{12}{35}+\frac{52}{15}-\pi-\frac{2}{3} = \frac{22}{7}-\pi$

41. $c^2 = a^2 + b^2 - 2bc \text{ Cos } C$

$c^2 = (a-b)^2 + (2-\sqrt{3})ab$

$(2x+1)^2 = (x+2)^2 + (2-\sqrt{3})(x^2-1)(x^2+x+1)$

$(2x+1)^2 - (x+2)^2 = (2-\sqrt{3})(x^2-1)(x^2+x+1)$

$3(x+1)(x-1) = (2-\sqrt{3})(x^2-1)(x^2+x+1)$

$x^2+x+1 = 3(2+\sqrt{3})$

$x^2+x-(5+3\sqrt{3}) = 0$

$\therefore x = \frac{-1\pm\sqrt{21+12\sqrt{3}}}{2}$

$= \frac{-1\pm\sqrt{(3+2\sqrt{3})^2}}{2}$

$= -(2+\sqrt{3})$ and $(1+\sqrt{3})$

But $x = -(2+\sqrt{3})$ c = 2x + 1 is negative

$\therefore x = 1+\sqrt{3}$ is the only possible value

Section III

Qns. 42, 43, 44

Let us take p = 5

$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, 3, 4\}\right\}$

Total number of matrices A that can be formed

$= 5^3 = 125$

Matrices are of the form

$\begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}, \begin{pmatrix} 1 & b \\ c & 1 \end{pmatrix}, \begin{pmatrix} 2 & b \\ c & 2 \end{pmatrix} \begin{pmatrix} 3 & b \\ c & 3 \end{pmatrix} \begin{pmatrix} 4 & b \\ c & 4 \end{pmatrix}$

where b c = 0, 1, 2, 3, 4

42. |A| will be of the form

0 – bc Oor (1 – bc) or (4 – bc)

Or (9 – bc)

Or (16 – bc)

Symmetric or skew symmetric matrices where |A| is divisible by 5 (=p)

are $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$

$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

There are 9 numbers

43. |A| divisible by p, (=5) but, trace of A is not divisible by p (=5)

Form

$\begin{pmatrix} 1 & \bullet \\ \bullet & 1 \end{pmatrix} \to 4$

$\begin{pmatrix} 2 & \bullet \\ \bullet & 2 \end{pmatrix} \to 4$

$\begin{pmatrix} 3 & \bullet \\ \bullet & 3 \end{pmatrix} \to 4$

$$\begin{pmatrix} 4 & \bullet \\ \bullet & 4 \end{pmatrix} \to 4$$

Total number = 16

44. Total number of matrices = 125

Consider all A^s whose determinants are divisible by p (=5)

= 16 + number of matrices of the form $\begin{pmatrix} 0 & \bullet \\ \bullet & 0 \end{pmatrix}$

= 16 + 9

= 25

Hence, the number of A in T_p such that det (A) not divisible by p (=5) is

= 125 – 25

= 100

[For p = 5, $(p^3 - 5p)$ also gives 100. However, when p = 3, the number of such matrices is verified as 18, which is obtained by setting p = 3

45. Any tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is of the form

$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} = 1$ —(1)

(1) is a tangent to the circle $(x - 4)^2 + y^2 = 16$

$$\frac{\frac{4\sec\theta}{3} - 1}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = \pm 4$$

$$\frac{(4\sec\theta - 3)^2}{9} \times \frac{36}{(4\sec^2 + 9\tan^2\theta)} = 16$$

$(4\sec\theta - 3)^2 = 4(4\sec^2\theta + 9\tan^2\theta)$

$16\sec^2\theta - 24\sec\theta + 9 = 16\sec^2\theta + 36\tan^2\theta$

$= 16\sec^2\theta + 36(\sec^2\theta - 1)$

$36\sec^2\theta + 24\sec\theta - 45 = 0$

$12\sec^2\theta + 8\sec\theta - 15 = 0$

$12\sec^2\theta + 18\sec\theta - 10\sec\theta - 15 = 0$

$(6\sec\theta)(2\sec\theta + 3) - 5(2\sec\theta + 3) = 0$

As $|\sec\theta| \geq 1$,

$\sec\theta = \frac{-3}{2}$ is the only solution

$\Rightarrow \tan\theta = -\frac{\sqrt{5}}{2}$

Equation of the common tangent is

$$\frac{x}{3} \times \frac{-3}{2} - \frac{y}{2} \times \frac{-\sqrt{5}}{2} = 1$$

$$\frac{-x}{2} + \frac{\sqrt{5}y}{4} = 1$$

$2x - \sqrt{5}\,y + 4 = 0$

46. Let a common point of intersection be $(3\sec\theta, 2\tan\theta)$

This point lies on the circle

$x^2 + y^2 = 8x = 0$

$9\sec^2\theta + 4\tan^2\theta - 24\sec\theta = 0$

$\Rightarrow 13s^2 - 24s - 4 = 0$

where s stands for $\sec\theta$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \tan\theta = \sqrt{3}$

Points A and B are $(6, 2\sqrt{3})$ and $(6, -2\sqrt{3})$

Required circle is

$(x - 6)^2 + y^2 - 12 = 0$

$x^2 + y^2 - 12x + 24 = 0$

Section IV

47. $C_1 \to C_1 + C_2 + C_3 \Rightarrow \begin{vmatrix} z & \omega & \omega^2 \\ z & z+\omega^2 & 1 \\ z & 1 & z+\omega \end{vmatrix} = 0$

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ we get

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & (z+\omega^2-\omega) & 1-\omega^2 \\ 0 & 1-\omega & z+\omega \end{vmatrix} = 0$$

i.e. $z\left[\left(z - (\omega - \omega^2)\right)\left(z + (\omega - \omega^2)\right) - (1-\omega)(1-\omega^2)\right] = 0$

$z\left[z^2 - (\omega - \omega^2)^2 - (1 - \omega^2 - \omega + 1)\right] = 0$

$z\left[z^2 - \omega^2(1 - 2\omega + \omega^2) - (2 - -1)\right] = 0$

$z\left[z^2 - \omega^2 . (-3\omega) + -3\right] = 0$

i.e. $z^3 = 0$

$\therefore$ number of z satisfying is 1

48. $\tan\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$

$\theta = n\pi + \frac{\pi}{2} - 5\theta$

$6\theta = 2\pi + \frac{\pi}{2}$

$\theta = \frac{n\pi}{6} + \frac{\pi}{12}, n = 0, \pm 1, \pm 2$ — (1)

Again, $\sin 2\theta = \cos 4\theta$

$= \sin\left(\frac{\pi}{2} - 4\theta\right)$

$2\theta = n\pi + (-1)^n\left(\frac{\pi}{2} - 4\theta\right)$

$$\theta = \frac{n\pi}{2} + \frac{(-1)^n\left(\frac{\pi}{2} - 4\theta\right)}{2}$$

$= \frac{n\pi}{2} + (-1)^n\left(\frac{\pi}{4} - 2\theta\right)$ — (2)

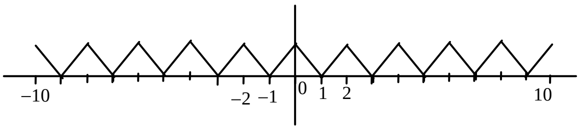
Common values of θ are

$\theta = \frac{\pi}{12}, \frac{-\pi}{4}, \frac{5\pi}{12}$

Number of solutions = 3

49. Here $f(x) = \begin{cases} \{x\} & \text{if } [x] \text{ is odd} \\ 1-\{x\} & \text{if } [x] \text{ is even} \end{cases}$

The graph of f(x) follows



$\therefore \frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x dx = \frac{\pi^2}{10}\times 10 \times \int_{-1}^{1} f(x)\cos\pi x dx$

$= \pi^2\left[\int_{-1}^{0}\{x\}\cos\pi x dx + \int_{0}^{1}(1-\{x\}\cos\pi x dx\right]$

y = –x, dy = –dx

{–x} = 1 – {x}

$\int_0^1 (1-\{x\})\cos\pi x dx = \int_0^{-1}(1-\{-y\}\cos\pi y\, dy$

$= -\int_0^{-1}(1-\{y\}\cos\pi y dy$

$= \int_{-1}^{0}\{y\}\cos\pi y\, dy$

$= \pi^2 . 2 \int_{-1}^{0} x\cos\pi x dx$

$= 2\pi^2 \int_{-1}^{0}\{x\}\cos\pi x dx$

$= 2\pi^2\left[\left(x.\frac{\sin\pi x}{\pi}\right)_{-1}^{0} - \int_{-1}^{0} 1\frac{\sin\pi x}{\pi}dx\right]$

$= 2\pi^2\left[\left(\frac{-1}{\pi}.\frac{-\cos\pi x}{\pi}\right)_0^1\right] = 4$

50. Required equation of plane is

$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

i.e., x – 2y + z = 0. This plane is parallel to the given plane Ax – 2y + z = d

$\therefore$ A = 1

$\therefore$ distance between the plane is

$\left|\frac{d}{\sqrt{1+4+1}}\right| = \sqrt{6} \Rightarrow d = 6$

51. 2x + y = 1 is a tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

i.e. $1 = 4a^2 - b^2$ —(1)

Since 2x + y = 1 intersects the directions

$x = \frac{a}{e}$ at x – axis implies

$\frac{a}{e} = \frac{1}{2}$ i.e. 2a = e —(2)

$\therefore 1 = e^2 - b^2 \Rightarrow b^2 = e^2 - 1$

$\Rightarrow a^2 (e^2 - 1) = (e^2 - 1)$

$\Rightarrow a^2 = 1$

a = 1 and e = 2

52. $S_k = \frac{a}{1-r} = \frac{\frac{k-1}{k1}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$

$\therefore \sum_{k=1}^{100}\left|\frac{k^2-3k+1}{(k-1)!}\right| = \sum_{k=1}^{100}\left|\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right|$

Note:

$k^2 - 3k + 1 = A(k-1)(k-2) + B(k-1) + C$

$k = 1 \Rightarrow C = -1$

$k = 2 \Rightarrow B = 0$

Equating coefficient of $k^2 \Rightarrow A = 1$

$\therefore$ Required summation

$= 0 - 1 + 1 - \frac{1}{2!} + \frac{1}{1!} - \frac{1}{3!} + \text{——} \frac{1}{97!} - \frac{1}{99!}$

$\therefore$ Answer $= \frac{100^2}{100!} + 1 - \frac{1}{98!} - \frac{1}{99!}$

$= \frac{100^2}{100!} + 1 - \frac{99\times100}{100!} - \frac{100}{100!}$

$= 1 + \frac{100^2 - 100^2}{100!} = 1$

53. $Y - y = \frac{dy}{dx}(X - x)$

X = 0

$Y = y - x\frac{dy}{dx}$

$y - x\frac{dy}{dx} = x^3$

$x\frac{dy}{dx} - y = -x^3$

$\frac{dy}{dx} - \frac{1}{x}y = -x^2$

$\frac{y}{x} = C - \int x^2 \times \frac{1}{x}dx \qquad = C - \frac{x^2}{2}$

$y = Cx - \frac{x^3}{2}$

y(1) = 1

$C - \frac{1}{2} = 1$

$C = \frac{3}{2}$

$y = \frac{3x}{2} - \frac{x^3}{2}$

$y(-3) = \frac{-9}{2} + \frac{27}{2} = \frac{18}{2} = 9$

54. $(\bar{a} \times \bar{b}) \times (\bar{a} - 2\bar{b})$

$= (\bar{a} \times \bar{b}) \times \bar{a} - 2(\bar{a} \times \bar{b}) \times \bar{b}$

$= (\bar{a} - \bar{a}) - (\bar{a}.\bar{b})\bar{a} - 2[(\bar{b} - \bar{a}) - (\bar{b}.\bar{b})\bar{a}]$

$= \bar{b} - (\bar{a}.\bar{b})\bar{a} - 2(\bar{a}.\bar{b})\bar{b} + 2\bar{a}$

$= (2\bar{a} + \bar{b})$ since $\bar{a} - \bar{b} = 0$

$\therefore\ (2\bar{a} + \bar{b})((\bar{a} \times \bar{b}) \times (\bar{a} - 2\bar{b}))$

$= (2\bar{a} + \bar{b})^2$

$= 4a^2 + b^2 = 5$

55. 2^{nd} equation is

xyz sin 3θ = 2z cos3θ + 2y sin 3θ

Substitute in the first equation

(y +z) cos3θ = 2zcos3θ + 2ysin3θ

y(cos3θ − 2sin3θ) = zcos3θ — (1)

(y + 2z) cos3θ + y sin3θ = (y + z) cos3θ

zcosθ + ysin3θ = 0 — (2)

From (1), (2) $\frac{y}{z} \Rightarrow \frac{\cos 3\theta}{\cos 3\theta - 2\sin 3\theta} = \frac{-\cos 3\theta}{\sin 3\theta}$

cos3θ = 0 OR

cos3θ − 2sin3θ = −sin3θ

cos3θ = sin3θ

tan3θ = 1

$3\theta = 2n\pi \pm \frac{\pi}{2}$ $\qquad$ $3\theta = n\pi \pm \frac{\pi}{4}$

$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ $\qquad$ $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$

n = 0 → 2

n = 1 → 3

n = 2 → 1

n = 3 → 1

Total number of solutions = 7

56. $y = \frac{2}{2\sin^2\theta + 3.2\sin\theta\cos\theta + 5.2\cos^2\theta}$

$= \frac{2}{1 + \cos 2\theta + 3.\sin 2\theta + 5(1 + \cos^2\theta)}$

$= \frac{2}{6 + 3\sin 2\theta + 4\cos^2\theta}$

$\therefore$ Maximum of $y = \frac{2}{6-5} = 2$

PART III

57	58	59	60	61	62	63	64
B	**C**	**A**	**D**	**C**	**A**	**D**	**C**

65	66	67	68	69
A, C	**A, D**	**A, B, C**	**A, B**	**A, C**

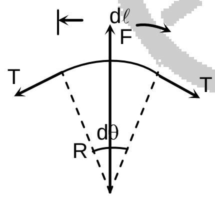
70	71	72	73	74
C	**B**	**D**	**A**	**B**

75	76	77	78	79	80	81	82	83	84
6	**6**	**4**	**5**	**9**	**3**	**4**	**4**	**8**	**3**

Section I

57. As ω increases, $X_C = \frac{1}{\omega c}$ decreases $\Rightarrow$ Z decreases $\Rightarrow$ I increases.

58.



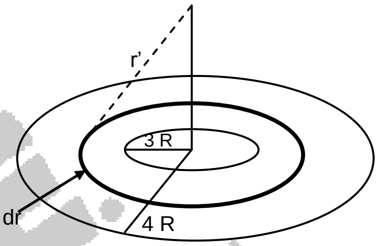
$F = IBd\ell = IBRd\theta$
Component of T, radial, will add upto
$2T\sin\frac{d\theta}{2} \cong T.d\theta$
(cosdθ components cancel)
$\therefore IBRd\theta = Td\theta \Rightarrow T = IBR = IB\frac{L}{2\pi}$

59. $P_1 + mg\mu\cos\theta = mg\sin\theta \Rightarrow P_1$ and f are in same direction. $\therefore$ P and f are in opposite direction. (In case P_1, f acts upward; In case of P_2, f acts downward).

60. Ideal gas assumes no intermolecular interaction i.e., at low pressures and high temperatures of real gas.

61. Formula:
$R = \frac{\rho\ell}{A}$.
Here $\ell = L$, $A = Lt$ $\therefore$ $R = \frac{\rho L}{Lt} = \frac{\rho}{t}$

62.



$$dV = -\frac{G\sigma 2\pi r dr}{r'}$$

$$= -\frac{G\sigma 2\pi 4R\tan\theta.4R\sec^2\theta d\theta}{4R\sec\theta}$$

$$= -G\sigma.2\pi.4R\frac{\sin\theta\ d\theta}{\cos^2\theta}$$

$$V = \int_{37°}^{45°} G\sigma 2\pi.4R\frac{\sin\theta d\theta}{\cos^2\theta}$$

$$= -\frac{GM}{7\pi R^2}.2\pi.4R\int_{37°}^{45°}\frac{\sin\theta d\theta}{\cos^2\theta}$$

Let $\cos\theta = t$
$-\sin\theta\ d\theta = dt$
Substituting

$$V = -\frac{GM}{7R}.8\int_{\frac{4}{5}}^{\frac{1}{\sqrt{2}}}\frac{(-dt)}{t^2}$$

$$= -\frac{8GM}{7R}.\left[\frac{1}{t}\right]_{\frac{4}{5}}^{\frac{1}{\sqrt{2}}}$$

$$= -\frac{8GM}{7R}.\left(\sqrt{2}-\frac{5}{4}\right)$$

$= -\frac{2GM}{7R}(4\sqrt{2}-5)$

$\therefore W = +\frac{2GM}{7R}(4\sqrt{2}-5)$

63. Since $R \propto \frac{1}{\text{Power}}$, $R_{100} < R_{60} < R_{40}$

$\Rightarrow \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$. This inequality holds at any temperature θ (assuming same material) because

$\frac{1}{R_{100}(1+\alpha\theta)} > \frac{1}{R_{60}(1+\alpha\theta)} > \frac{1}{R_{40}(1+\alpha\theta)}$.

64. Ammeter is a galvanometer with a low shunt resistance. Voltmeter is a galvanometer with a high resistance in series. Only option C meets the requirement.

Section II

65. Apply formulae for v_1 and v_2 for elastic collision.

$(v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$ and similarly for $v_2)$

$\Rightarrow v_1 = -2 \text{ m s}^{-1}$ gives

$u_1 = +3 \text{ m s}^{-1}$ and using this $u_2 = 1 \text{ m s}^{-1}$

$\therefore$ B is incorrect.

$v_{cm} = \frac{1 \text{ kg} \times 3 \text{ m s}^{-1}}{6 \text{ kg}} = \frac{1}{2} \text{ m s}^{-1}$

v_{cm} is unchanged.

$\therefore K.E = \frac{1}{2} \times 6 \times \left(\frac{1}{2}\right)^2 = \frac{3}{4}$ J

$\Rightarrow$ C is incorrect.

Total K.E = $\frac{1}{2} \times 1 \times 3^2 = 4.5$ J

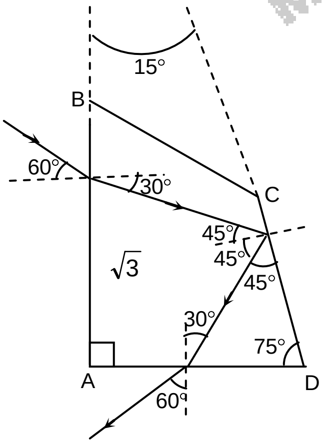
$\Rightarrow$ D is incorrect.

66. Direction of lines of force show Q_1 +ve and Q_2 –ve density of lines around Q_1 > that around Q_2

$\therefore |Q_1| > |Q_2|$

$\therefore$ Null point can only be on the right side of Q_2.

67.



$i_1 = 60°$, $n = \sqrt{3} \Rightarrow r_1 = 30°$

$r_2 - r_1 = A = 15°$ (formula for incident ray as shown)

$\Rightarrow r_2 = 45°$

$\Rightarrow$ internally reflected at 45°

(Since $\sin C = \frac{1}{\sqrt{3}} \Rightarrow C < 45°$)

From geometry incident angle on AD is 30°

$\Rightarrow$ emergent angle = 60°

$\therefore$ Angle between incident and emergent rays is the same as the angle between the two faces = 90°.

68. AB: Isothermal, BC: isobaric, CA: isochoric.

$\therefore$ A) correct, $W_{AB} = nRT_0 \ell n \frac{4V_0}{V_0} = P_0V_0 \ell n4$,

B) correct: **Note** : C) and D) are correct only if BC passes through the origin

69. Least count 1 s $\Rightarrow \frac{\Delta T}{T} = \frac{1 \text{ s}}{40 \text{ s}} = 0.025$

$\therefore T = \frac{40 \text{ s}}{20 \text{ cycles}} = 2 \text{ s} \Rightarrow$

$\Delta T = 0.025 \times T = 0.05$ s

$g = 4\pi^2 \frac{\ell}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{2\Delta T}{T} = 0.05 = 5\%$

Section III

70. $E < V_0$ so that motion is periodic

Clearly $E > 0$ ($\because$ At $x = 0$, $V = 0$ and K.E > 0)

71. [Use dimensional analysis]

B is the only correct option.

72. For $|x| > x_0$, $V = V_0$ = constant

$\Rightarrow F = -\frac{dV}{dx} = 0 \Rightarrow$

no acceleration.

73. Superconducting transition temperature falls when magnetic field is increased.

74.

Field	T_C
0	100 K
7.5 T	75 K

$\therefore$ As field varies from zero to 7.5 T, T_C variation is from 100 K to 75 K, which clearly shows for a field of 5 T, T_C must be in between 75 K and 100 K.

Section IV

75. Let angular velocity be ω. Let distances from CM be d_B and d_A. Then the required ratio is

$$\frac{m_A d_A^2\omega + m_B d_B^2\omega}{m_B d_B^2\omega}$$

But $m_A d_A = m_B d_B$ ($\because$ CM definition)

$\therefore$ Ratio = $\frac{d_A + d_B}{d_B} = \frac{d_A}{d_B} + 1 = \frac{m_B}{m_A} + 1 = 6$

76. Using $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

(i) $-\frac{1}{(-25)} + \frac{1}{v} = \frac{1}{20} \Rightarrow v = 100 \Rightarrow m_{25} = -4$

(ii) $-\frac{1}{(-50)} + \frac{1}{v'} = \frac{1}{20} \Rightarrow v' = \frac{100}{3} \Rightarrow m_{50} = -\frac{2}{3}$

$\therefore \frac{m_{25}}{m_{50}} = 6$

77. $\omega = \sqrt{\frac{k}{m}}$ where $k = \frac{AY}{\ell}$

$\Rightarrow \omega = \frac{AY}{\ell m} \Rightarrow 140 = \sqrt{\frac{4.9\times10^{-7}\times n\times10^{9}}{1\times0.1}}$

$\Rightarrow n = 4$

78. $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$

$= -3\cos(2x - 6t)$

$Y = y_1 + y_2 \Rightarrow$ amplitude $A = \sqrt{3^2 + 4^2} = 5$

79. From Wien's law: $\frac{T_1}{T_2} = \frac{1500}{500} = 3$

Energy radiated from surface = $4\pi r^2.\ \sigma T^4$

$\therefore$ Ratio of energy emitted = $\left(\frac{r_1}{r_2}\right)^2.\left(\frac{T_1}{T_2}\right)^4$

$= \left(\frac{1}{3}\right)^2.3^4 = 9$

80. $g = \frac{GM}{R^2} \propto \rho R \qquad \therefore \frac{g_p}{g_e} = \left(\frac{\rho_p}{\rho_e}\right)\left(\frac{R_p}{R_e}\right)$

$\Rightarrow \frac{R_p}{R_e} = \frac{\sqrt{6}}{11}.\frac{3}{2}$

$v_{esc} = \sqrt{2\ gR} \quad \therefore \frac{v_{esc}^p}{v_{esc}^e} = \sqrt{\frac{g_p R_p}{g_e R_e}}$

$= \sqrt{\frac{\sqrt{6}}{11}.\frac{\sqrt{6}}{11}.\frac{3}{2}}$

$= \sqrt{\frac{9}{121}} = \frac{3}{11}$

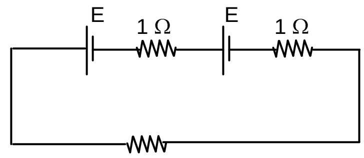
$\Rightarrow v_{esc}^p = 3$ km s^{-1}

81. $\left(1 + \frac{v_2}{c}\right) - \left(1 - \frac{v_1}{c}\right)$ is given as 0.012

$\Rightarrow v_2 - v_1 = 0.012 \times 330 \cong 4$

82.

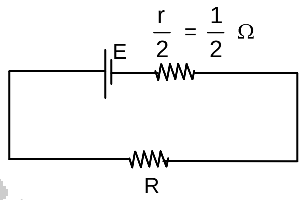
Series



$\rightarrow i_1^2 R = J_1$

$\therefore \left(\frac{2E}{R+2}\right)^2 R = J_1$

Parallel



$\rightarrow \left(\frac{E}{R + \frac{1}{2}}\right)^2 R = J_2$

$\frac{J_1}{J_2} = 2.25 = \frac{4}{(R+2)^2}\left(R + \frac{1}{2}\right)^2 \Rightarrow R = 4\ \Omega$

83. $420 = \frac{m}{1000} \times (0 - (-5)) \times 2100 + \frac{1}{1000} \times 3.36\times10^5$

$\Rightarrow m = 8$

84. $\frac{p^2}{2m} = qV;\ \frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p}$

$\left(\frac{p_\alpha}{p_p}\right)^2 = \left[\frac{m_\alpha}{m_p}.\frac{q_\alpha}{q_p}\right] = [4 \times 2] = 8$

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{8} = 2\sqrt{2}$