P1-10-6-0
494020

PAPER -1

Code 0

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This Question Paper containing 32 pages having 84 questions.
2. The question paper CODE is printed on the right hand top corner of this sheet and also on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed.
5. The answer sheet, a machine–readable Objective Response Sheet (ORS), is provided separately.
6. Do not Tamper / mutilate the **ORS** or the booklet.
7. Do not break the seals of the question-paper booklet before instructed to do so by the invigilators.

B. Filling the ORS

8. The **ORS** has **CODE** printed on its lower and upper parts.
9. Make sure the **CODE** on the **ORS** is the same as that on this booklet. **If the codes do not match, ask for a change of the Booklet.**
10. Write your Registration No., Name and Name of center and sign with pen in appropriate boxes. **Do not write these anywhere else.**
11. Darken the appropriate bubbles below your registration number with **HB Pencil**.

C. Question paper format and **Marking Scheme :**

12. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists four Sections.
13. For each questions in **Section I**, you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other case **minus one (–1) mark** will be awarded
14. For each question in **Section II**, you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. Partial marks will be awarded for partially correct answers. **No negative marking** will be awarded in this section.
15. For each question in **Section III**, you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
16. For each question in **Section IV**, you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. **No negative marks** will be awarded for in this Section.

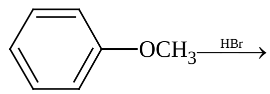
PART I : CHEMISTRY

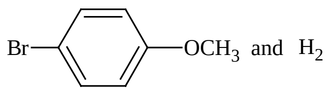
PAPER - I

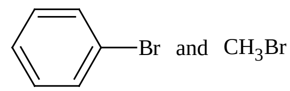
SECTION – I

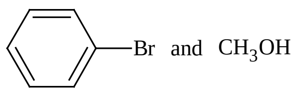
Single Correct Choice Type

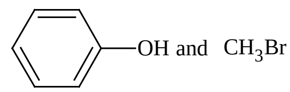
This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. In the reaction  the products are

(A) 

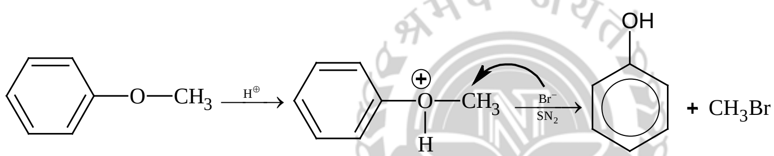
(B) 

(C) 

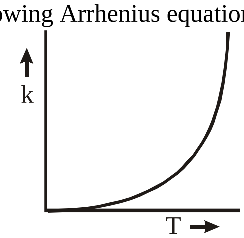
(D) 

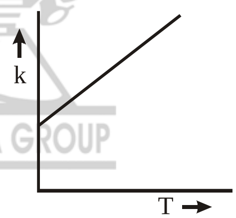
Key: **(D)**

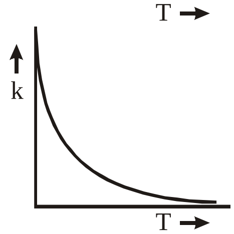
Sol.: $HBr \longrightarrow H^+ + Br^-$

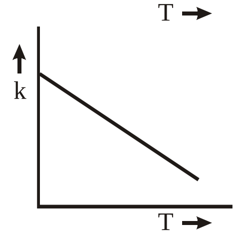


2. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that following Arrhenius equation is

(A) 

(B) 

(C) 

(D) 

Key: **(A)**

Sol.: $K = Ae^{\frac{-E_a}{RT}}$

Rate constant K increases exponentially with the rise in temperature. Since rate const. K also depends upon orientation factor A hence its maximum value is not at all infinity rather limited to an optimal value.

3. The species which by definition has ZERO standard molar enthalpy of formation at 298 K is

(A) Br_2 (g)　　(B) Cl_2 (g)

(C) H_2O (g)　　(D) CH_4(g)

Key: **(B)**

Sol.: Bromine and water exist in liquid state at 298 K. Methane is not an elemental species.

4. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is

(A) $[Cr(H_2O)_4(O_2N)]Cl_2$ (B) $[Cr(H_2O)_4Cl_2](NO_2)$

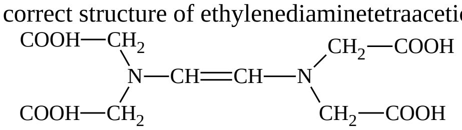
(C) $[Cr(H_2O)_4Cl(ONO)]Cl$ (D) $[Cr(H_2O)_4Cl_2(NO_2)]\cdot H_2O$

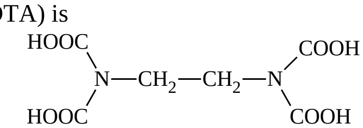
Key: **(B)**

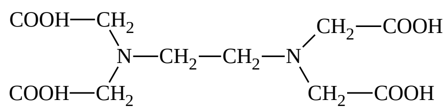
Sol.: $[Cr(H_2O)_4Cl(NO_2)]Cl \xrightleftharpoons{\text{ionization}} [Cr(H_2O)_4Cl(NO_2)]^+ + Cl^-$

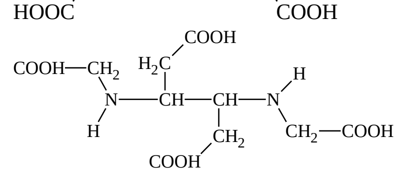
$[Cr(H_2O)_4Cl_2](NO_2) \xrightleftharpoons{\text{Ionization}} [Cr(H_2O)_4Cl_2]^+ + \bar{N}O_2$.

5. The correct structure of ethylenediaminetetraacetic acid (EDTA) is

(A) 

(B) 

(C) 

(D) 

Key: **(C)**

Sol.: Based on facts

6. The bond energy (in kcal mol^{-1}) of a C—C single bond is approximately

(A) 1 (B) 10

(C) 100 (D) 1000.

Key: **(C)**

Sol.: C – C single bond dissociation energy ranges between 88 to 150 K cal mol^{-1}.

7. The synthesis of 3-octyne is achieved by adding a bromolkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$ (B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C\equiv CH$

(C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C\equiv CH$ (D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$.

Key: **(D)**

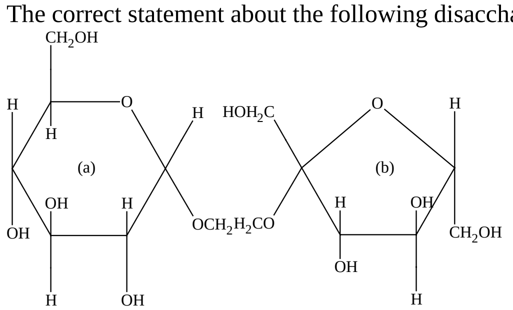
Sol.: $CH_3 – CH_2 – C \equiv C – C – CH_2 – CH_2 – CH_3$

3 – octyne

$$CH_3-CH_2-C\equiv CH \xrightarrow{NaNH_2} CH_3-CH_2-C\equiv \overset{\ominus}{C} \xrightarrow{CH_3-CH_2-CH_2-CH_2-Br} CH_3-CH_2-CH_2-CH_2-C\equiv C-CH_2-CH_3$$

3 octyne

8. The correct statement about the following disaccharide is



(A) Ring (a) is pyranose with α-glycosidic link (B) Ring (a) is furanose with α-glycosidic link

(C) Ring (b) is furanose with α-glycosidic link (D) Ring (b) is pyranose with β-glycosidic link.

Key: **(A)**

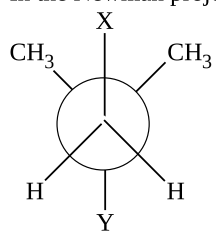
Sol.: Ring (a) is pyranose whereas ring(b) is furanose. α-anomeric form of ring (a) is attached through glycosidic bond.

SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

9. In the Newman projection for 2, 2-dimethylbutane



X and Y can respectively be

(A) H and H

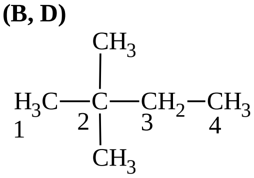
(B) H and C_2H_5

(C) C_2H_5 and H

(D) CH_3 and CH_3.

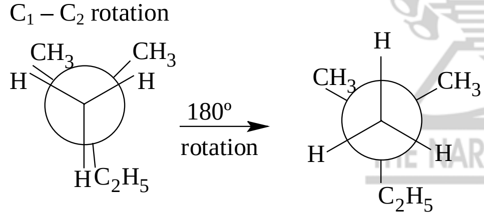
Key: **(B, D)**

Sol.:

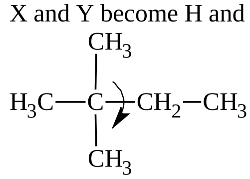


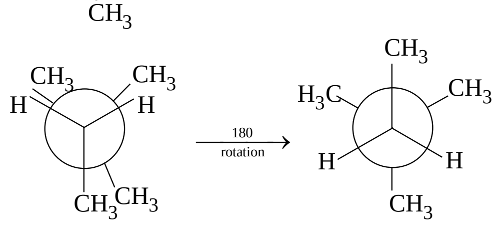
2, 2-dimethyl butane

$C_1 – C_2$ rotation



X and Y become H and C_2H_5





X and Y become CH_3 and CH_3.

10. The regent(s) used for softening the temporary hardness of water is (are)

(A) $Ca_3(PO_4)_2$

(B) $Ca(OH)_2$

(C) Na_2CO_3

(D) $NaOCl$.

Key: **(B, C)**

Sol.: Temporary hardness is due to the presence of bicarbonates of Ca and Mg. Temporary hardness can be removed by clarke's process which involves the addition of slaked lime

$$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 + 2H_2O$$

Washing soda removes both the temporary and permanent hardness.

$$Ca(HCO_3)_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaHCO_3.$$

11. Among the following, the intensive property is (properties are)

(A) molar conductivity (B) electromotive force

(C) resistance (D) heat capacity.

Key: **(A)**

Sol.: EMF, resistance and heat capacity are extensive properties ofcourse, resistivity is an intensive property.

12. Aqueous solutions of HNO_3, KOH, CH_3COOH, and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which form a buffer upon mixing is(are)

(A) HNO_3 and CH_3COOH (B) KOH and CH_3COONa

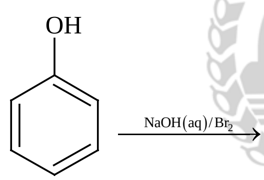
(C) HNO_3 and CH_3COONa (D) CH_3COOH and CH_3COONa .

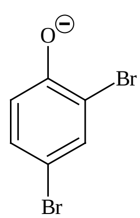
Key: **(C, D)**

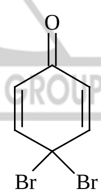
Sol.: Mixture of weak acid and its salt are known as acidic buffer.

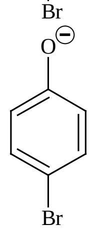
$$\underset{\text{strong acid}}{H^+NO_3^-} + CH_3CO_2^-Na^+ \longrightarrow \underset{\text{weak acid}}{CH_3CO_2H} + Na^+NO_3^-$$

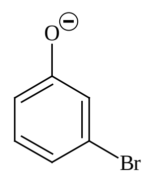
In an acid-based reaction. The equilibrium shifts to the direction which results in the formation of weaker acid.

13. In the reaction  the intermediate(s) is (are)

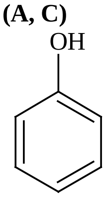
(A) 

(B) 

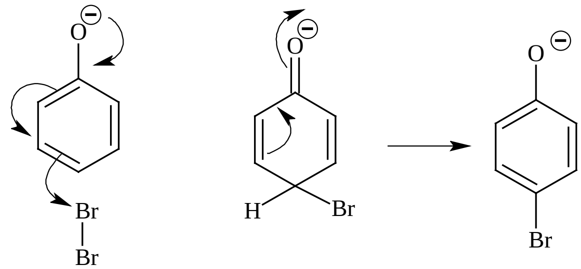
(C) 

(D) 

Key: **(A, C)**

Sol.: 

is strongly activating towards EAS reaction and it is ortho-para directing



SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 14 to 16

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4 .\ 5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

14. Partial roasting of chalcopyrite produces

(A) Cu_2S and FeO (B) Cu_2O and FeO

(C) CuS and Fe_2O_3 (D) Cu_2O and Fe_2O_3

Key: **(B)**

Sol: $CuFeS_2 + O_2 \rightarrow Cu_2S + 2FeS + SO_2$

The sulphites of copper and iron are partially oxidized

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$

$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$.

15. Iron is removed from chalcopyrite as

(A) FeO (B) FeS

(C) Fe_2O_3 (D) $FeSiO_3$

Key: **(D)**

Sol: Fe is removed in the form of $FeSiO_3$.

$FeO + SiO_2 \longrightarrow FeSiO_3$

16. In self-reduction, the reducing species is

(A) S (B) O^{2-}

(C) S^{2-} (D) SO_2

Key: **(C)**

Sol: $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

S^{-2} oxidized into S^{+4} hence it is reducing species .

Paragraph for Questions 17 to 18

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is :

$$M(s) \mid M^+(aq;\ 0.05\ molar) \parallel M^+(aq;\ 1molar) \mid M(s)$$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

17. For the above cell

(A) $E_{cell} < 0;\ \Delta G > 0$ (B) $E_{cell} > 0;\ \Delta G < 0$

(C) $E_{cell} < 0;\ \Delta G^\circ > 0$ (D) $E_{cell} > 0;\ \Delta G^\circ < 0$

Key: **(B)**

Sol: $E_{cell} = \frac{-2.303RT}{F}\log\frac{0.05}{1}$ = a positive value

= 70 mV (given)

Hence $\Delta G < 0$.

18. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be

(A) 35 mV (B) 70 mV

(C) 140 mV (D) 700 mV

Key: **(C)**

Sol: $E_{cell} = \frac{-2.303RT}{F}\log\frac{0.0025}{1}$

$= \frac{-2.303RT}{F}\log(0.05)^2$

= 2 × 70 mV

= 140 mV.

SECTION – IV
Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

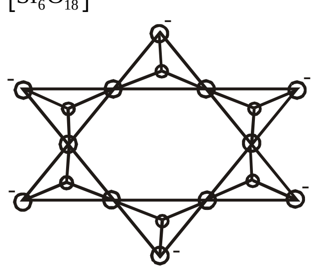
19. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

Key: **(3)**

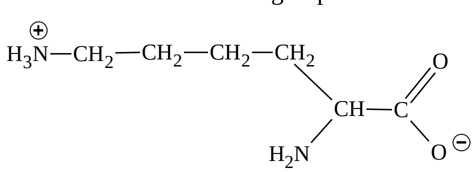
Sol.: $[Si_6O_{18}]^{12-}$

$Be_3Al_2Bi_6O_{18}$

$[Si_6O_{18}]^{12-}$

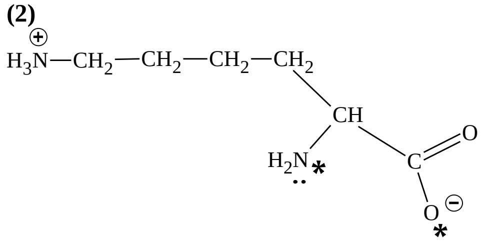


20. The total number of basic groups in the following form of lysine is



Key: **(2)**

Sol.:

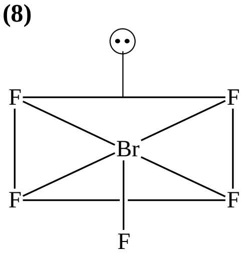


* Group are basic.

21. Based on VSEPR theory, the number of 90 degree F – Br – F angles in BrF_5 is

Key: **(8)**

Sol.:



The structure of BrF_5 is square pyramidal. The number of FBrF angles having the value of 90º is eight (8). Due to trivial distortion, however, the bond angles (F—Br—F) are slightly less than 90º(85º).

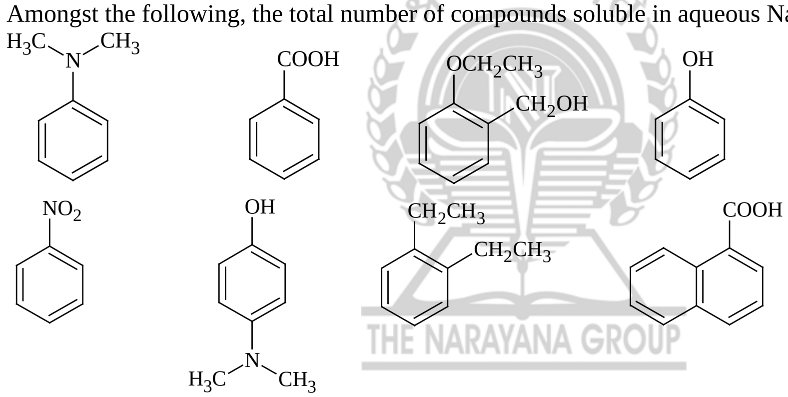
22. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

KCN	K_2SO_4	$(NH_4)_2C_2O_4$	NaCl	$Zn(NO_3)_2$
$FeCl_3$	K_2CO_3	NH_4NO_3	LiCN	

Key: **(3)**

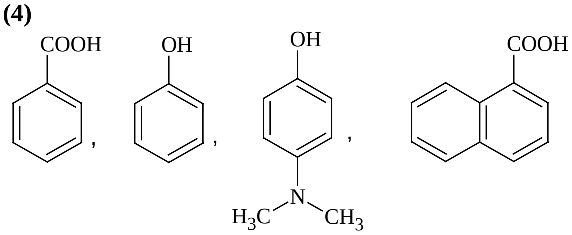
Sol.: KCN, K_2CO_3, LiCN are basic salt can convert red litmus to blue.

23. Amongst the following, the total number of compounds soluble in aqueous NaOH is



Key: **(4)**

Sol.:



are soluble in aq. NaOH.

24. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is

Key: **(3)**

Sol.: Average = $= \frac{25.2 + 25.25 + 25.0}{3}$

$= 75.45 / 3$

$= 25.15 \approx 25.1.$

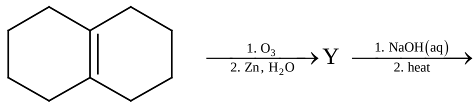
No. of significant figure = 3.

25. The number of neutrons emitted when ${}^{235}_{92}U$ undergoes controlled nuclear fission to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ is

Key **(3)**

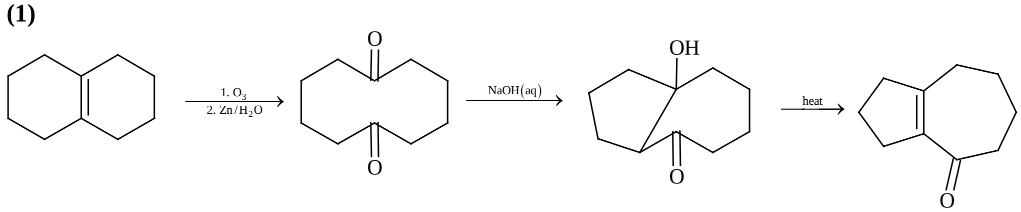
Sol.: ${}^{235}_{92}U \longrightarrow {}^{142}_{54}Xe + {}^{90}_{38}Sr + 3{}^{1}_{0}n$.

26. In the scheme given below, the total number of intramolecular aldol condensation products formed from 'Y' is



Key: **(1)**

Sol.:



27. The concentration of R in the reaction R → P was measured as a function of time and the following data is obtained :

[R] (moloar)	1.0	0.75	0.40	0.10
t (min.)	0.0	0.05	0.12	0.18

The order of the reaction is

Key: **(0)**

Sol.: $R \longrightarrow P$

$-\frac{dc}{dt}$ after 0.05 min = $\frac{0.25}{0.05} = 5 \;\; M\ min^{-1}$

$-\frac{dc}{dt}$ after 0.12 min = $\frac{060}{0.12} = 5 \;\; M\ min^{-1}$

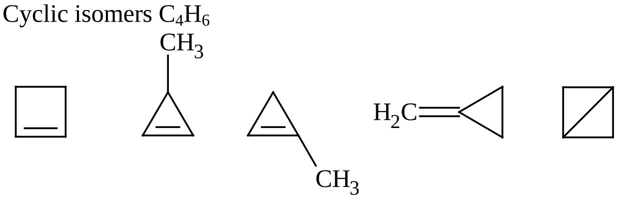
$-\frac{dc}{dt}$ after 0.18 min = $\frac{90}{0.18} = 5\ M\ min^{-1}$

The average rate remains same throughout. This implies that rate is independent of concentration (zero order).

28. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is

Key **(5)**

Sol.: Cyclic isomers C_4H_6



Total isomers = 5

PART II: MATHEMATICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

29. The number of 3 × 3 matrices A whose are either 0 or 1 and for which the system $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has exactly two distinct solutions, is

(A) 0
(B) $2^9 - 1$
(C) 168
(D) 2

Key (A)

Sol. Three planes cannot meet only at two distinct points. Hence 'A' is correct.

30. The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}\,dt$ is

(A) 0
(B) $\frac{1}{12}$
(C) $\frac{1}{24}$
(D) $\frac{1}{64}$

Key (B)

Sol. $$\lim_{x\to 0}\frac{\int_0^x \frac{t\ln(1+t)\,dt}{t^4+4}}{x^3}$$

$$=\lim_{x\to 0}\frac{x\ln(1+x)}{(x^4+4)3x^2} = \lim_{x\to 0}\frac{\ln(1+x)}{3x(x^4+4)}$$

$$= \frac{1}{4\times 3}\lim_{x\to 0}\frac{\ln(1+x)}{x} = \frac{1}{12}.$$

31. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Key (B)

Sol. $$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)\{(\alpha + \beta)^2 - 3\alpha\beta\}$$

$$q = -p(p^2 - 3\alpha\beta)$$

$$\Rightarrow q + p^3 = 3\alpha\beta p \Rightarrow \alpha\beta = \frac{(q+p^3)}{3p}$$

$$\alpha^2 + \beta^2 = p^2 - 2\frac{(q+p^3)}{3p} = \frac{3p^3-2q-2p^3}{3p} = \frac{p^3-2q}{3p}$$

$$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{p^3-2q}{q+p^3}$$

$\Rightarrow x^2 - \frac{(p^3-2q)}{p^3+q}x+1 = 0 \Rightarrow (p^3 + q)x^2 - (p^3 - 2q)\, x + (p^3 + q) = 0$

32. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$
(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Key (C)

Sol. $\vec{n}=(3\hat{i}+4\hat{j}+2\hat{k})\times(4\hat{i}+2\hat{j}+3\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 2\\ 4 & 2 & 3\end{vmatrix} = 8\hat{i}-\hat{j}-10\hat{k}$

The equation of plane containing the IInd and IIIrd given lines.

$\vec{r}.(8\hat{i}-\hat{j}-10\hat{k}) = 0 \Rightarrow 8x - y - 10z = 0.$

Now normal vector to the required plane is given by

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 8 & -1 & -10\end{vmatrix} = -26\hat{i}+52\hat{j}-26\hat{k}$$

$= -26(\hat{i}-2\hat{j}+\hat{k})$

The equation of the required plane is $x - 2y + z = 0$.

33. If the angle A, B and C of the triangle are in the an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$
(C) 1 (D) $\sqrt{3}$

Key (D)

Sol. $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A = \frac{\sin A}{\sin C}2\sin C\cos C+\frac{\sin C}{\sin A}2\sin A\cos A$

$= 2\sin(A + C)$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

34. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2}+e^{-x^2}$, $g(x) = xe^{-x^2}$ and $h(x) = x^2 + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) $a = b$ and $c \neq b$ (B) $a = c$ and $a \neq b$
(C) $a \neq b$ (D) $a = b = c$

Key (D)

Sol. $1 \geq x \geq x^2 \quad \forall x \in [0, 1]$

$e^{x^2} \geq xe^{x^2} \geq x^2e^{x^2} \quad \forall x \in [0, 1]$

i.e., $e^{-x^2}+e^{x^2} \geq e^{-x^2}+xe^{x^2} \geq e^{-x^2}+x^2e^{x^2}$

equality holds when $x = 1$

i.e., $f(x) \geq g(x) \geq h(x)\ \forall x \in [0, 1]$

Hence a = b = c.

35. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1 , r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Key (C)

Sol. Required prob. = $\frac{2\times2\times2(3!)}{6\times6\times6} = \frac{2}{9} \Rightarrow \frac{2}{9}$

36. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square (C) rectangle, but not a square

(D) rhombus, but not a square

Key (A)

Sol. $P: -2\hat{i} - \hat{j}$, $Q: 4\hat{i}$, $R: 3\hat{i} + 3\hat{j}$, $S: -3\hat{i} + 2\hat{j}$

$\overrightarrow{PQ} =$ of P $= 6\hat{i} + \hat{j}$

$\overrightarrow{QR} = 3\hat{i} + 3\hat{j} - 4\hat{i} = -\hat{i} + 3\hat{j}$

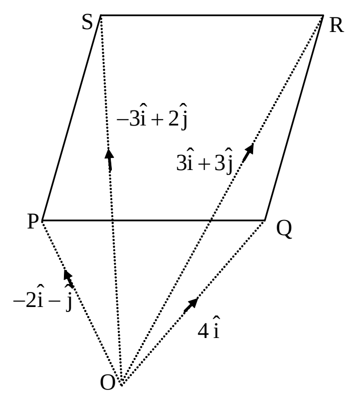
$\overrightarrow{PS} = -3\hat{i} + 2\hat{j} + 2\hat{i} + \hat{j} = -\hat{i} + 3\hat{j}$

$\overrightarrow{SR} = 3\hat{i} + 3\hat{j} + 3\hat{i} - 2\hat{j} = 6\hat{i} + \hat{j}$

$\overrightarrow{PQ}.\overrightarrow{PS} = (6\hat{i} + \hat{j}).(-\hat{i} + 3\hat{j}) = -3 \neq 0$

Here $\overrightarrow{PQ} \parallel \overrightarrow{SR}$ and $\overrightarrow{PS} \parallel \overrightarrow{QR}$

but $\overrightarrow{PQ}$ is not perpendicular to $\overrightarrow{PS}$



SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

37. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t) z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ (B) Arg $(z - z_1)$ = Arg $(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$ (D) Arg $(z - z_1)$ = Arg $(z_2 - z_1)$

Key (A, C, D)

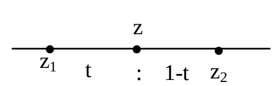
Sol. As $z = (1 - t) z_1 + tz_2$

$\Rightarrow z_1, z, z_2$ are collinear

$\therefore$ A, D are correct

Also $\frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1}$

$\therefore$ (C) is correct.



38. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$

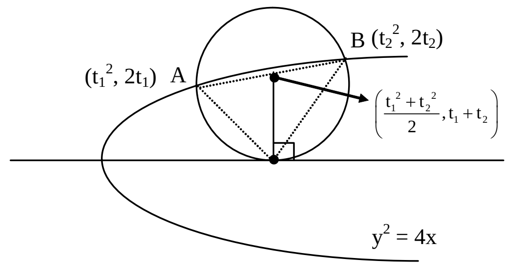
(C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Key (C, D)

Sol. Slope of line AB

$$M = \frac{(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)} = \left(\frac{2}{t_1 + t_2}\right) = \pm\frac{2}{r}$$

As $|t_1 + t_2| = r$



39. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Key (B, C)

Sol. $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt, \ x > 0$

$f'(x) = \frac{1}{x} + \sqrt{1+\sin x}, \ x > 0$

Clearly $f'(x)$ exists for all $x \in (0, \infty)$

and $f'(x)$ is continuous on $(0,\infty)$

but not differentiable on $(0,\infty)$

More over $f'(x), f(x) > 0 \ \forall x \in (1, \infty)$

and $\ln x + \int_0^x \sqrt{1+\sin t}\, dt > \frac{1}{x} + \sqrt{1+\sin x} \ \ \forall x \in (\pi, \infty)$

$\frac{1}{x}$ is not bounded.

$\therefore$ (D) is incorrect .

Hence, option B, C are correct.

40. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, dx$ is (are)

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$

(C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

Key (A)

Sol. $\int_0^1 \frac{x^4(1-x)^4}{(1+x^2)}\, dx$

$\int_0^1 (x^6 - 4x^5 + 5x^4)dx - \int_0^1 \frac{4x^4}{1+x^2}dx \ = \frac{10}{21} - 4\int_0^1 \frac{x^4 - 1 + 1}{x^2 + 1}dx$

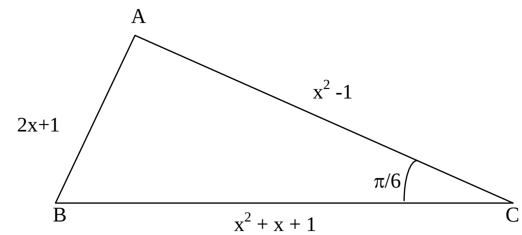
$= \frac{22}{7} - \pi$

41. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Key (B)



Sol. $\frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2 + (x^2-1)^2 - (2x+1)^2}{2.(x^2-1)(x^2+x+1)}$

$\frac{\sqrt{3}}{2} = \frac{(x^2+3x+2)(x^2-x)+(x^2-1)^2}{2(x^2-1)(x^2+x+1)}$

$\frac{\sqrt{3}}{2} = \frac{(x+2)x + x^2 - 1}{2(x^2+x+1)}$

$\sqrt{3} = \frac{2x^2+2x-1}{x^2+x+1}$

$\Rightarrow x^2\left(\sqrt{3}-2\right) + x\left(\sqrt{3}-2\right) + \sqrt{3}+1 = 0$

$x = \frac{-(\sqrt{3}-2) \pm \sqrt{(\sqrt{3}-2)^2 - 4(\sqrt{3}-2)(\sqrt{3}+1)}}{2(\sqrt{3}-2)} = \sqrt{3}+1.$

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 42 to 44

Let p be an odd prime number and T_p be the following set of 2×2 matrices.

$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, ..., p-1\} \right\}$

Sol. 42 to 44

as A is symmetric b = c

det A = $a^2 - b^2 = (a + b)(a - b)$

a, b, c, $\in$ {0, 1, 2, ….. p – 1}

no. of numbers of type

np = 1

np + 1 = 1

np + 2 = 1 $n \in I$

⋮

np + (p – 1) = 1

42. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) 2 (p – 1)

(C) $(p-1)^2 + 1$ (D) 2p – 1

Key (D)

Sol. as det(A) is div. by p $\Rightarrow$ either a + b div. by p corresponding nu. Of ways = (p – 1) [excluding zero]

or (a – b) is div. by p corresponding no. of ways = p

Total number of ways = 2p – 1

43. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is
[Note: The trace of a matrix is the sum of its diagonal entries.]
(A) (p – q) (p^2 p p + q) (B) $p^3 - (p-1)^2$
(C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

Key (C)

Sol. as Tr(A) not div. by p $\Rightarrow a \neq 0$
det(A) is div. by p $\Rightarrow a^2 - bc$ div. by p
no. of ways of selection of a, b, c
$(p-1)[(p-1) \times 1] = (p-1)^2$

44. The number of A in T_p such that det (A) is not divisible by p is
(A) $2p^2$ (B) $p^3 - 5p$
(C) $p^3 - 3p$ (D) $p^3 - p^2$

Key (D)

Sol. Total number of A = $p \times p \times p = p^3$
No. of A such that det(A) div. by p
$= (p-1)^2$ + no. of A in which a = 0
$= (p-1)^2 + p + p - 1$
$= p^2$
required no. = $p^3 - p^2$.

Paragraph for Questions Nos. 45 to 46

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is
(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$
(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Key (B)

Sol. Equation of tangent at point P(θ)

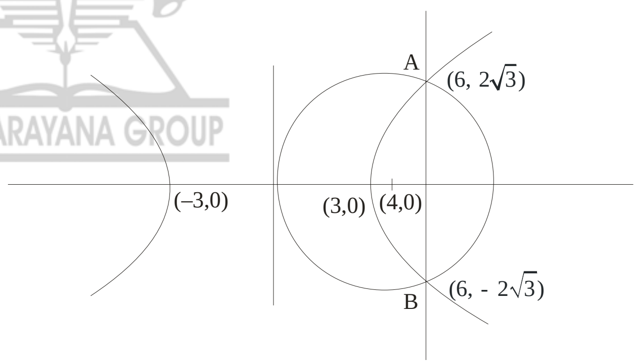
$$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} - 1 = 0 \quad \text{.....(i)}$$

since eq. (i) will be a tangent to the circle

$$\therefore \frac{\frac{4\sec\theta}{3} - 1}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = 4$$

by solving it we will get

$$2x - \sqrt{5}y + 4 = 0$$



46. Equation of the circle with AB as its diameter is
(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$
(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

Key (A)

Sol. $\dfrac{x^9}{9} = 1 + \dfrac{(-x^2 + 8x)}{4}$

$4x^2 = 36 + 9(-x^2 + 8x)$

$13x^2 - 72x - 36 = 0$

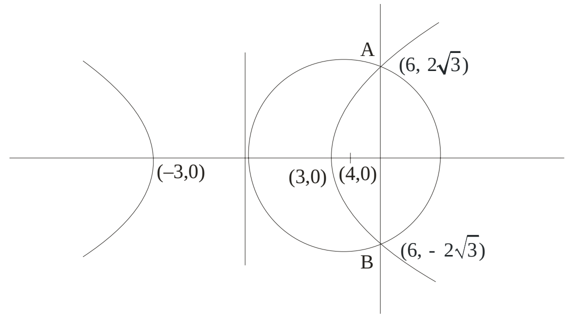
$x = 6,$

$y = \pm 2\sqrt{3}$

Required equation of circle is

$(x - 6)^2 + y^2 - 12 = 0$

$x^2 + y^2 - 12x + 24 = 0$



SECTION – IV

Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

47. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \text{ is equal to}$$

Key (1)

Sol. $\omega = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$

ω is one of cube root of unity.

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \qquad \left[\because 1+\omega+\omega^2 = 0\right]$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$ gives

$$\begin{vmatrix} 0 & 0 & z \\ \omega - z - \omega^2 & z + \omega^2 - 1 & 1 \\ \omega^2 - 1 & 1 - z - \omega & z + \omega \end{vmatrix} = 0$$

$z\left[(\omega - z - \omega^2)(1 - z - \omega) - (\omega^2 - 1)(z + \omega^2 - 1)\right] = 0$

$z[z^2] = 0$

$\Rightarrow \quad z^3 = 0$

$= z = 0$

Ans. is $= 1$

48. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ± 1, ± 2 and tanθ = cot5θ as well as sin2θ = cos4θ is

Key (3)

Sol. $\tan\theta = \cot 5\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$

$\theta = n\pi + \frac{\pi}{2}0 - 5\theta$

$6\theta = n\pi + \frac{\pi}{2}$

$\theta = \frac{n\pi}{6} + \frac{\pi}{12} n \in I$ …..(i)

$\sin 2\theta = \cos 4\theta$

$\sin 2\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow \quad 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow \quad 2\sin^2 2\theta + 2\sin 2\theta - \sin 2\theta - 1 = 0$

$(2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$

$\sin 2\theta = \frac{1}{2}, \qquad \sin 2\theta = -1$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \qquad 2\theta = -\frac{\pi}{6}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12} \qquad \theta = -\frac{\pi}{4}$

All three values of θ which satisfy the eq. (i).

49. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [-10, 10] by

f(x) = $\begin{cases} x - [x] & \text{if } [x] \text{is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x \; dx$ is

Key (0)

Sol. Case-I :

When f(x) is odd

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx = 0$$

Case-II :

When f(x) is even

$$\frac{\pi^2}{10}\int_{-10}^{10} (1+[x]-x)\cos \pi x dx = \frac{\pi^2}{10}\left\{\int_{-10}^{10} (1+[x])\cos \pi x dx - \int_{-10}^{10} x\cos \pi x dx\right.$$

$$= \frac{\pi^2}{10}\int_{-10}^{10} (1+[x])\cos \pi x dx = 0$$

50. If the distance between the plane Ax – 2y + z = d and the plane containing the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

Key (6)

Sol. The equation of the plane containing the given lines will be a(x – 1) + b(y – 2) + c(z – 3) = 0 where a, b, c are direction ratios of normal to the plane considering vectors parallel to the two lines

2i + 3j + 4k and 3i + 4j + 5k

So $2a_1 + 3b_1 + 4c_1 = 0$

$3a_1 + 4b_1 + 5c_1 = 0$

$$\frac{a_1}{15-16} = \frac{-b_1}{10-12} = \frac{c_1}{8-9}$$

So the plane is x – 2y + z = 0

Hence distance between two planes

$$\frac{|d|}{\sqrt{1^2+2^2+1}}=\sqrt{6}$$
$$|d|=6$$

51. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Key (2)

Sol. Since the line 2x + y – 1 = 0 is tangent

so, $C^2 = a^2m^2 - b^2$

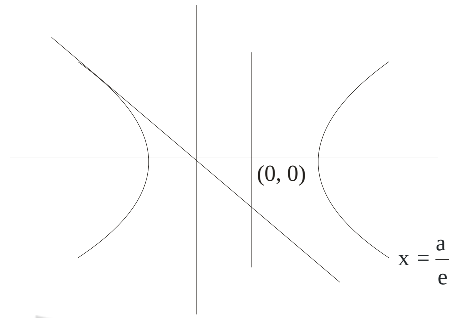
$1 = 4a^2 - b^2$ …..(i)

Also line passes through $\left(-\frac{a}{e},0\right)$

So, $2\left(-\frac{a}{e}\right)=1$

$4a^2 = e^2$ ……(ii)

Using (i) and (ii) e = 2



52. Let S_k, k = 1, 2, ... , 100, denote the sum of the infinite geometric series whose whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$ is

Key (1)

Sol. $$S_k=\frac{\frac{k-1}{k!}}{1-\frac{1}{k}}=\frac{1}{(k-1)!}$$

Now $$\sum_{k=1}^{100}\left|(k^2-3k+1)\frac{1}{(k-1)!}\right|$$

$$=\sum_{k=1}^{100}\left|\frac{(k-1)^2-k}{(k-1)!}\right|$$

$$=\sum_{k=1}^{100}\left|\frac{(k-1)}{(k-1)!}-\frac{k}{(k-1)!}\right|$$

Putting the values of k

$$1-\frac{100}{99!}$$

$$=1-\frac{(100)^2}{(100)!}$$

So, $$\frac{(100)^2}{(100!)}+1-\frac{(100)^2}{(100)!}=3$$

53. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(-3) is equal to

Key (9)

Sol. eq. of tangent at P(x, y)

$$Y - y = \frac{dy}{dx}(X - x)$$

y-integer $y - x\frac{dy}{dx} = x^3$

$$\frac{dy}{dx} - \frac{y}{x} = -x^2$$

I.F. = $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

The solution

$$y \times \frac{1}{x} = \int -x^2 \times \frac{1}{x}dx$$

$$\frac{y}{x} = -\frac{x^2}{2} + C$$

$$f(1) = 1 \quad \Rightarrow \quad C = \frac{3}{2}$$

$$f(x) = y = \frac{3x - x^3}{2}$$

$$f(-3) = 9$$

54. If $\bar{a}$ and $\bar{b}$ are vectors in space given by $\bar{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\bar{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is

Key (5)

Sol. $|\bar{a}| = |\bar{b}| = 1 \qquad \bar{a}.\bar{b} = 0$

Let $\vec{l} = (\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} - 2(\vec{a} \times \vec{b}) \times \vec{b}$

$= |a|^2\ \vec{b} - (\vec{a}.\vec{b})\vec{a} - 2(a.b)\vec{b} + 2\,|b|\,a.$

$= \vec{b} + 2\vec{a}$

$(2\vec{a} + \vec{b}).\vec{l} = |2\vec{a} + \vec{b}|^2 = 5$

55. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

(y + z) cos3θ = (xyz) sin 3θ

xsin3θ = $\frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

(xyz) sin3θ = (y + 2z) cos3θ + ysin 3θ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

Key (3)

Sol. $(y+z)\cos 3\theta = (xyz)\sin 3\theta$ (A)

$x\sin 3\theta = \frac{2\cos 3\theta}{y} = \frac{2\sin 3\theta}{z}$ (B)

$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$ (C)

$$\tan 3\theta = \frac{y+z}{xyz} = \frac{2z}{y(xz-2)} = \frac{y+2z}{xyz-y}$$

as $y \neq 0$

$(y + z)(xz - 2) = 2z(xz)$

$xyz + xz^2 - 2z - 2y = 2xz^2$

$xyz = 2y + 2z + xz^2$ …..(i)
$2z(xz – 1) = (y + 2z)(xz – 2)$
$2xz^2 – 2z = xyz + (z^2 – 4z – 2y)$
$xyz = 2y + 2z$ …..(ii)
$xz^2 = 0$
$\Rightarrow$ $x = 0$ as $z \neq 0$
from (i) $(y + z)\cos 3\theta = 0$
$\Rightarrow$ $y + z = 0$ or $\cos 3\theta = 0$
but when $\cos 3\theta = 0$ from (B)
$\sin 3\theta = 0$ not possible
So $y = –z$ putting in (B) and (C)
$x = 0$
$\sin 3\theta = \cos 3\theta$
$\Rightarrow$ $\tan 3\theta = 1 \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

56. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Key (2)

Sol. Let $y = \frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} = \frac{1}{3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta}$

$$-\frac{5}{2} \leq 2\cos 2\theta + \frac{3}{2}\sin 2\theta \leq \frac{5}{2}$$

max. value of $y = \frac{1}{3 - \frac{5}{2}} = 2$

PART III: PHYSICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

57. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer (B) the bulb glows brighter
(C) total impedance of the circuit is unchanged (D) total impedance of the circuit increases.

Key. (B)

Sol. $P = V_{rms} I_{rms} \cos\phi$

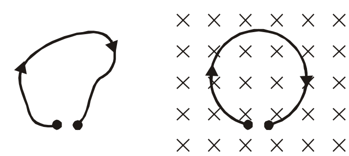
$$= \frac{V_{rms}^{2}}{Z} \cdot \frac{R}{Z}$$

$$= \frac{V_{rms}^{2}R}{Z^2}$$

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

As ω increase Z, decreases, so P increases.
Hence correct option is (B).

58. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is

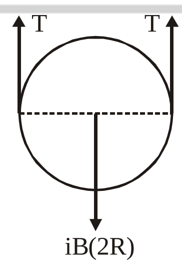


(A) IBL (B) $\frac{IBL}{\pi}$

(C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$.

Key. (C)

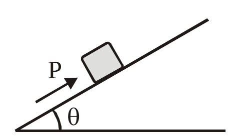
Sol. $2T = iB(2R)$



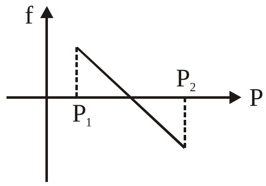
$$T = \frac{iBL}{2\pi}$$

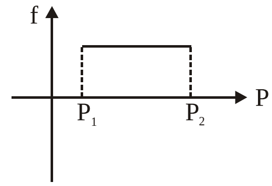
Hence correct option is (C).

59. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to P_2 mg $(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like

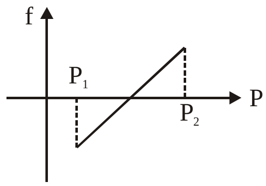


(A) (B)

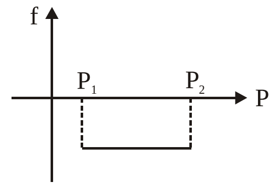




(C)



(D)



Key. (A)

Sol. $P_1 = mg(\sin\theta - \mu\cos\theta)$

friction$_{initial}$ = μmgcosθ up along the plane

friction$_{final}$ = μmgcosθ down along the plane

Hence correct option is (A).

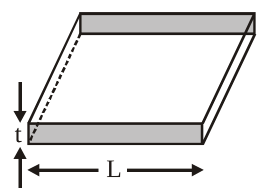
60. A real gas behaves like an ideal gas if its

(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high.

Key. (D)

Sol. For ideal gas behaviour pressure should be low and temperature should be high.
Hence correct option is (D).

61. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



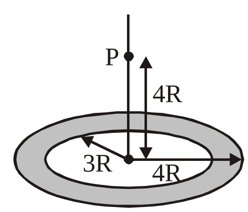
(A) directly proportional to L
(B) directly proportional to t
(C) independent of L
(D) independent of t.

Key. (C)

Sol. $R = \rho\dfrac{\ell}{A} = \rho\cdot\dfrac{L}{Lt} = \dfrac{\rho}{t}$

Hence correct option is (C).

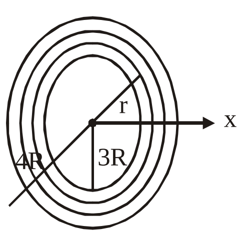
62. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



(A) $\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$
(B) $-\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$
(C) $\dfrac{GM}{4R}$
(D) $\dfrac{2GM}{5R}\left(\sqrt{2}-1\right)$.

Key. (A)

Sol.



$$dV = -\frac{G \cdot \sigma \cdot 2\pi r dr}{\sqrt{r^2 + x^2}}$$

$$V = -2\pi G\sigma \int_{3R}^{4R} \frac{rdr}{\sqrt{r^2 + x^2}}$$

$$r^2 + x^2 = z$$

$$2r\,dr = dz$$

$$\int \frac{rdr}{\sqrt{r^2 + x^2}} = \frac{dz}{2\sqrt{z}}$$

$$= \frac{1}{2}\frac{z}{1/2} = \sqrt{z}$$

$$V = -2\pi G\sigma \left[\sqrt{r^2 + x^2}\right]_{3R}^{4R}$$

$$= -2\pi Gr\left[4R\sqrt{2} - 5\right]$$

$$W = (1)\left[0 + 2\pi G\sigma(4R\sqrt{2} - 5R)\right]$$

$$= 2\pi G \cdot \frac{M}{\pi(16-9)R^2}(4R\sqrt{2} - 5R)$$

$$= \frac{2\pi GM}{7R}(4\sqrt{2} - 5)\ .$$

Hence correct option is (A).

63. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$

(B) $R_{100} = R_{40} + R_{60}$

(C) $R_{100} > R_{60} > R_{40}$

(D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$.

Key. (D)

Sol. $R = \frac{v^2}{P}$

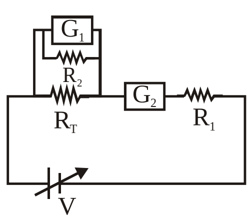
As temperature increase, resistance increases

So, $R_{40} > R_{60} > R_{100}$.

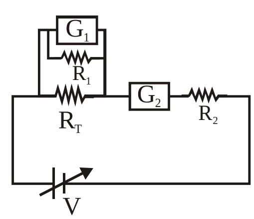
Hence correct option is (D).

64. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

(A)

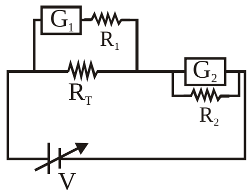


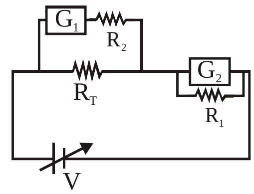
(B)



(C)

(D)





Key. (C)

Sol. An ideal voltmeter should have large resistance and an ideal ammeter should have low resistance.
Hence correct option is (C).

SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

65. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

(A) total momentum of the system is 3 kg ms^{-1}
(B) momentum of 5 kg mass after collision is 4 kg ms^{-1}
(C) kinetic energy of the center of mass is 0.75 J
(D) total kinetic energy of the system is 4 J.

Key. (A), (C)

Sol. $(1)(V) + (5)(0) = (1)(-2) + 5V'$

$V = 5V' - 2$ …(i)

$$\frac{V'+2}{V-0} = 1$$

$V' = V - 2$ …(ii)

$V = 5(V - 2) - 2$

From equation (i) and (ii)

$V = 5V - 10 - 2$

$4V = 12$

$V = 3$ m/s.

$P_i = (1)(3) = 3$ kg – m/s.

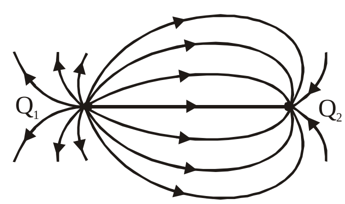
$$V_{CM} = \frac{(1)(3)+(5)(0)}{6} = \frac{1}{2}\,m/s$$

$$K_{CM} = \frac{1}{2}(6)\frac{1}{4} = \frac{3}{4} = 0.75\ J$$

$$K_{total} = \frac{1}{2}(1)(3)^2 = 4.5\ J$$

Hence correct options are (A), (C).

66. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x–axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$
(B) $|Q_1| < |Q_2|$
(C) at a finite distance to the left of Q_1 the electric field is zero
(D) at a finite distance to the right of Q_2 the electric field is zero.

Key. (A), (D)

Sol. Density of field lines is more are Q_1

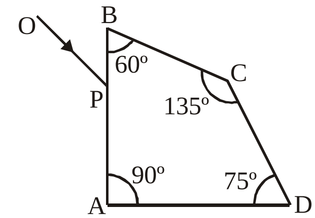
$\therefore$ $|Q_1| > |Q_2|$

Q_1 and Q_2 are of opposite signs

So, null point will be closer to charge of smaller magnitude i.e., Q_2

Hence correct options are (A), (D).

67. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60º (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ?



(A) the ray gets totally internally reflected at face CD
(B) the ray comes out through face AD
(C) the angle between the incident ray and the emergent ray is 90º
(D) the angle between the incident ray and the emergent ray is 120º.

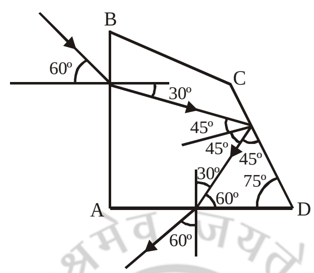
Key. (A), (B), (C)

Sol. $1\sin 60 = \sqrt{3}\sin r$

$r = 30^\circ$

$\sin\theta_C = \frac{1}{\sqrt{3}}$

$\theta_C \simeq 35^\circ$

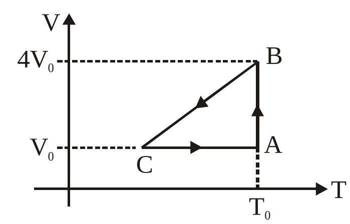


At CD angle of incidence is greater than θ_C.
At AD angle of incidence is less than critical angle
So ray will come out of AD.
Angle of deviation

$-30 + 90 + 30 = 90^\circ$

Hence correct options are (A), (B), (C)

68. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in figure. Its pressure at A is P_0. Choose the correct option(s) from the following :



(A) internal energies at A and B are the same
(B) work done by the gas is process AB is $P_0V_0\ell n\,4$
(C) pressure at C is $\frac{P_0}{4}$
(D) temperature at C is $\frac{T_0}{4}$.

Key. (A), (B), (C), (D)

Sol. Internal energy of an ideal gas depends on temperature

$W_{BC} = nRT\,\ell n\frac{V_2}{V_1}$

$= (1)(R)\frac{P_0V_0}{R}\ell n\frac{4V_0}{V_0}$

$= P_0V_0\ell n4$

For CA

$\frac{P}{T} = \text{constant}$

P at C $= \frac{P_0}{4}$

T at C $= \frac{T_0}{4}$

Hence all options are correct.

69. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 second fore this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?
(A) error ΔT in measuring T, the time period, is 0.05 seconds
(B) error ΔT in measuring T, the time period, is 1 second
(C) percentage error in the determination of g is 5%
(D) percentage error in the determination of g is 2.5%.

Key. (A), (C)

Sol. Error in measurement of $T = \frac{1}{20} s = 0.05\ s$

$$\frac{dg}{g} = 2\frac{dT}{T}$$

$$\frac{dg}{g} = 2 \times \frac{1}{40}$$

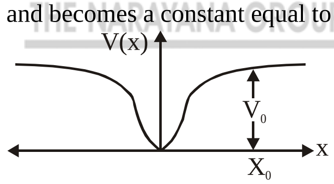
% error in calculation of g = 5%.

SECTION – III
Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 70 to 72

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



70. If the total energy of the particle is E, it will perform periodic motion only if
(A) $E < 0$
(B) $E > 0$
(C) $V_0 > E > 0$
(D) $E > V_0$.

Key. (B)

Sol. For periodic motion
Total energy should be less than V_0 but greater than zero.
Hence (C) is correct.

71. For periodic motion of small amplitude A, the time period T of this particle is proportional to
(A) $A\sqrt{\frac{m}{\alpha}}$
(B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
(C) $A\sqrt{\frac{\alpha}{m}}$
(D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$.

Key. (B)

Sol. Dimensionally only B is correct.

72. The acceleration of this particle for $|x| > X_0$ is

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$

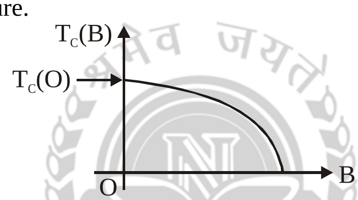
(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) zero.

Key. (D)

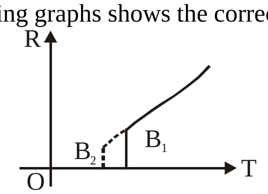
Sol. For $x > x_0$
potential energy is constant
force on particle is zero.
Hence (D) is correct.

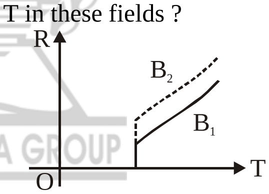
Paragraph for Question Nos. 73 to 74

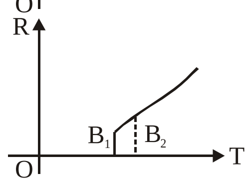
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

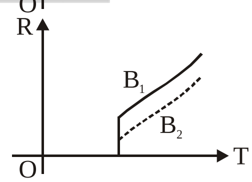


73. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic field B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields ?

(A)


(B)


(C)


(D)


Key. (A)

Sol. As B increases, critical temperature decreases.

74. A superconductors has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(A) B = 5 Tesla, $T_C(B) = 80$ K (B) B = 5 Tesla, $75 \text{ K} < T_C(B) < 100 \text{ K}$

(C) B = 10 Tesla, $75 \text{ K} < T_C(B) < 100 \text{ K}$ (D) B = 10 Tesla, $T_C(B) = 70$ K.

Key. (B)

SECTION – IV

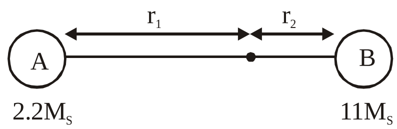
Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

75. A binary star consists of two starts A (mass 2.2. M_S) and B (mass 1 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their center of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the center of mass is.

Key. 6.

Sol.



$$\frac{11r_2^2 + 2.2r_1^2}{11r_2^2}$$

$$11\,r_2 = 2.2\,r_1$$

$$= 1 + \frac{2.2}{11}\cdot\frac{r_1^2}{r_2^2}$$

$$= 1 + \frac{2.2}{11}\times\left(\frac{11}{2.2}\right)^2 = 6\,.$$

76. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Key. 6.

Sol. $$m = \frac{|f|}{|f - u|}$$

$$\frac{m_{25}}{m_{50}} = 6\,.$$

77. A 0.1 kg mass is suspended form a wire of negligible mass. The length of the wire is 1 m and its cross–sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

Key. 4.

Sol. $$\omega^2 = \frac{K}{m}$$

$$140\times140 = \frac{YA}{\ell m} = \frac{Y(4.9\times10^{-7})}{(1)(0.1)}$$

$$140\times140 = y(49)\times10^{-7}$$

$$y = 4\times10^9$$

n = 4.

78. When two progressive waves y_1 4 sin (2x – 6t) and y_2 = 3 $\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Key. 5.

Sol. Amplitude $= \sqrt{4^2 + 3^2} = 5\,.$

79. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of total energy radiated by A to that of B ?

Key. 9.

Sol. (T_1) (500 nm) = T_2 (1500 nm)

$T_1 = 3T_2$

$E_A = \sigma \cdot 4\pi(6\text{cm})^2 (T_1)^4$

$E_B = \sigma \cdot 4\pi(18\text{cm})^2 (T_1)^4$

$\frac{E_A}{E_B} = \left(\frac{1}{3}\right)^2 \times (3)^4 = 9$.

80. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

Key. 2.

Sol. $\frac{GM_P}{R_P^2} = \frac{\sqrt{6}}{11} g = \frac{\sqrt{6}}{11}\frac{GM_e}{R_e^2}$ …(i)

$\sqrt{2g_eR_e} = 11 \text{ km/s}$

$\sqrt{2g_PR_P} = x$

$\frac{g_eR_e}{g_PR_P} = \frac{(11)^2}{x^2}$

$\frac{\frac{GM_e}{R_e^2}\cdot R_e}{\frac{GM_P}{R_P^2}\cdot R_P} = \frac{121}{x^2}$

$\frac{\frac{M_e}{R_e}}{\frac{M_P}{R_P}} = \frac{121}{x^2}$ …(ii)

$\frac{M_P}{R_P^3} = \frac{2}{3}\cdot\frac{M_e}{R_e^3}$ …(iii)

x = 2.

81. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Key. 7.

Sol. $f_1 = \frac{V + V_{C_1}}{V - V_{C_1}} f_0$

$f_2 = \frac{V + V_{C_2}}{V - V_{C_2}} f_0$

$\Delta f = \left[\frac{V + V_{C_1}}{V - V_{C_1}} - \frac{V + V_{C_2}}{V - V_{C_2}}\right] f_0 = \frac{1.2}{100} f_0$

$= \frac{2\Delta V_C}{V} f_0 = \frac{1.2 f_0}{100}$

$\Delta V_C = 7 \text{ km/hr}$.

82. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is

Key. 4.

Sol.
$$J_1 = \left(\frac{2\varepsilon}{2+R}\right)^2 R$$
$$J_2 = \left(\frac{\varepsilon}{0.5+R}\right)^2 R$$
$$2.25 = \frac{4(0.5+R)^2}{(2+R)^2}$$
$$\frac{9}{4} = \frac{4(R+0.5)}{2+R}$$
$$\frac{3}{2} = \frac{2R+2}{2+R}$$
$$6+3R = 4R+2$$
$$R = 4\Omega.$$

83. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5ºC at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Key. 8.

Sol.
$$\left[m(2100)(5)+1(3.36\times10^5)\right]\times10^{-3} = 420$$
$$11m+336 = 420$$
$$11m = 420-336$$
$$= 84$$
$$m = 8\ \text{gm}.$$

84. An α–particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

Key. 3.

Sol.
$$\lambda = \frac{h}{\sqrt{2mk}}$$
$$k = qV$$
$$\lambda = \frac{h}{\sqrt{2(m)qV}}$$
$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\lambda}{m_p q_p}} = \sqrt{\frac{(4m)(2q)}{(m)q}} = 2\sqrt{2} = 3.$$