AIEEE 2009 — Mathematics

61. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c+1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$$, then the value of n is

(1) zero (2) any even integer
(3) any odd integer (4) any integer

61. **(3)** Since, two row interchanges are required.

62. If the mean deviation of the numbers 1, $1 + d$, $1 + 2d$, ..., $1 + 100\,d$ from their mean is 255, then the d is equal to

(1) 10.0 (2) 20.0 (3) 10.1 (4) 20.2

62. **(3)** Mean = $1 + 50\,d = x_M$

$\therefore$ Mean deviation = $(1 / 101)\, \Sigma\, |x_M - x_i|$

$$= \frac{1}{101}\sum_{k=0}^{100} |(k-50)d| = ((50 \times 51) / 100)\, d = 255 \text{ for } d = 10.1.$$

63. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(1) greater than $4ab$ (2) less than $4ab$
(3) greater than $-4ab$ (4) less than $-4ab$

63. **(3)** $c^2 < 4ab$ and minimum value of expr. $= -c^2 > -4ab$.

64. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(1) **A** is false and **B** is false (2) **A** is false and **B** is true
(3) both **A** and **B** are true (4) both **A** and **B** are false

64. **(3)** Given relation : $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$.

65. The lines $p(p^2 + 1)\,x - y + q = 0$ and $(p^2 + 1)^2\,x + (p^2 + 1)y + 2q = 0$ are perpendicular to the common line for

(1) no value of p (2) exactly one value of p
(3) exactly two values of p (4) more than two values of p

65. **(2)** Slopes must be equal $\Rightarrow$ $p = -1$.

66. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(1) $A = B$ (2) $A = C$ (3) $B = C$ (4) $A \cap B = \phi$

66. **(3)** If $B \neq C$, then there should be some element in B (or C) for which (both) given statement can't be correct simultaneously.

67. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for

(1) exactly one value of (p, q)

(2) exactly two values of (p, q)

(3) more than two but not all values of (p, q)

(4) all values of (p, q)

67. **(1)** $3p^2 - pq + 2q^2 = 0 \Rightarrow (p, q) \equiv (0, 0)$.

68. Let $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

(1) $(6, -17)$ (2) $(-6, 7)$ (3) $(5, -15)$ (4) $(-5, 5)$

68. **(2)** $2\alpha + \beta = -5$ and $\alpha = -6 \Rightarrow \beta = 7$.

69. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(1) less than 500 (2) at least 500 but less than 750

(3) at least 750 but less than 1000 (4) at least 1000

69. **(4)** ${}^6C_4 \times 4! \times {}^3C_1 \times 1! = 1080$.

70. $\int_0^{\pi} [\cot x]dx$, where **[.]** denotes the greatest integer function, is equal to

(1) $\pi / 2$ (2) 1 (3) -1 (4) $-\pi / 2$

70. **(4)** $2I = \int_0^{\pi} [t] + [-t]dx$, $t = \cot x \Rightarrow 2I = \int_0^{\pi} -1dx \Rightarrow I = -\pi / 2$.

71. For real x, let $f(x) = x^3 + 5x + 1$, then

(1) f is one-one but not onto **R** (2) f is onto **R** but not one-one

(3) f is one-one and onto **R** (4) f is neither one-one nor onto **R**

71. **(3)** $f'(x) > 0$ and range is **R**.

72. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, If the probability of at least one success is greater then or equal to (9 / 10), then n is greater then

(1) $\frac{1}{\log_{10}^{4}-\log_{10}^{3}}$ (2) $\frac{1}{\log_{10}^{4}+\log_{10}^{3}}$

(3) $\frac{9}{\log_{10}^{4}-\log_{10}^{3}}$ (4) $\frac{4}{\log_{10}^{4}-\log_{10}^{3}}$

72. **(1)** $(1-p)^{n} \leq (1/10)$ $\Rightarrow$ $(4/3)^{n} \geq 10$ $\Rightarrow$ $n \geq \frac{1}{\log 4 - \log 3}$.

73. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1,1) for:

(1) all values of p (2) all except one value of p

(3) all except two values of p (4) exactly one value of p

73. **(2)** $S_1 + \lambda S_2 = 0$ $\Rightarrow$ $\lambda \neq -1$ $\Rightarrow$ $p \neq -1$.

74. The projections of a vector on the three coordinate axis are 6,– 3, 2 respectively. The direction cosines of the vector are:

(1) 6, –3, 2 (2) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (3) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (4) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$

74. **(3)** $\sqrt{(36 + 9 + 4)} = 7$ $\Rightarrow$ $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$.

75. If $\left|Z - \frac{4}{z}\right| = 2,$ then the maximum value of $|Z|$ is equal to

(1) $\sqrt{3} + 1$ (2) $\sqrt{5} + 1$ (3) 2 (4) $2 + \sqrt{2}$

75. **(2)** $2 \geq |Z| - (4/|z|)$ $\Rightarrow$ $|Z| \leq \sqrt{5} + 1$.

76. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to (1 / 3) . Then the circumcentre of the triangle ABC is at the point

(1) (0, 0) (2) $\left(\frac{5}{4}, 0\right)$ (3) $\left(\frac{5}{2}, 0\right)$ (4) $\left(\frac{5}{3}, 0\right)$

76. **(2)** Equation of circumcircle is $x^2 + y^2 - (5/2)\,x + 1 = 0$.

77. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

(1) 0 (2) 2 (3) 7 (4) 8

77. **(2)** $(9-1)^{2n} - (9.7-1)^{2n+1} = (-1)^{2n} - (-1)^{2n+1} = 2$. (mod 9).

78. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0) . Then the equation of the ellipse is

(1) $x^2 + 16y^2 = 16$ (2) $x^2 + 12y^2 = 16$
(3) $4x^2 + 48y^2 = 48$ (4) $4x^2 + 64y^2 = 48$

78. **(2)** $(x^2 / 16) + (y^2 / b^2) = 1 \Rightarrow (2, 1) \Rightarrow b^2 = 4/3 \Rightarrow x^2 + 12y^2 = 16^2$.

79. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$is

(1) 2 (2) 3 (3) 4 (4) 6

79. **(2)** Using formula for AGP, Sum = 3.

80. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is

(1) $y' = y^2$ (2) $y'' = y'y$ (3) $yy'' = y'$ (4) $yy'' = (y')^2$

80. **(4)** $y_1 = c_2 y \Rightarrow y_2 = (y_1 / y) \cdot y_1 \Rightarrow y_2 y = (y_1)^2$.

81. One ticket is selected at randon from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals:

(1) $\frac{1}{14}$ (2) $\frac{1}{7}$ (3) $\frac{5}{14}$ (4) $\frac{1}{50}$

81. **(1)** Fav. case = 08 and sample space **:** {00, 01, ..., 09, 10, 20, 30, 40}.

$\therefore \quad p = \frac{1}{14}$.

82. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(1) – 1 (2) 1 (3) log 2 (4) – log 2

82. **(1)** $x = 1 \Rightarrow \cot y = 0 \Rightarrow y_1 = -1$.

83. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is :

(1) 3 (2) 6 (3) 9 (4) 12

83. **(3)** Taking area w.r.t. y -axis (with adding and substracting triangle area)
$= 6 + 4 - 1 = 9$.

84. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval [–1, 1] :

(1) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

(2) $P(-1)$ is not minimum but $P(1)$ is the maximum of P

(3) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P

(4) neither$P(-1)$ is the minimum nor $P(1)$ is the maximum of P

84. **(2)** $x = 0$ is the only point of the minima and graph decreases from (– 1, 0) and then increases from (0, 1).

85. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is :

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$ (3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$

85. **(1)** $y_1 = 1$ $\Rightarrow$ (1 / 4, 1 / 2) is the closest point on the parabola from the line.

Directions: Questions number 86 to 90 are Assertion - Reason type questions. Each of these questions contains two statements**:**

Statement - 1 (Assertion) and

Statement - 2 (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Let $f(x) = (x + 1)^2 - 1, x \geq -1$.

Statement - 1: The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$

Statement - 2: f is a bijection.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.

(3) Statement - 1 is true, Statement - 2 is false.

(4) Statement - 1 is false, Statement - 2 is true.

86. **(1) or (2)** Both statements are correct and 1 can be proved using 2 (does not follow directly).

87. Let $f(x) = x \mid x \mid$ and $g(x) = \sin x$.

Statement - 1: gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement - 2: gof is twice differentiable at $x = 0$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.

(3) Statement - 1 is true, Statement - 2 is false.

(4) Statement - 1 is false, Statement - 2 is true.

87. **(3)** $h(x) = gof(x)$ $\Rightarrow$ $h''(0^+) = 2, h''(0^-) = -2$.

88. **Statement - 1:** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement - 2: The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is false.
(4) Statement - 1 is false, Statement - 2 is true.

88. **(4)** Variance = $(n^2 - 1) / 3$.

89. **Statement - 1:** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement - 2: $\sim (p \leftrightarrow \sim q)$ is a tautology.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is false.
(4) Statement - 1 is false, Statement - 2 is true.

89. **(3)** Statement reduces to $(p \wedge q) \vee (\sim p \wedge \sim q)$, which is not a tautology.

90. Let A be a 2×2 matrix.
Statement - 1: adj (adj (A)) = A.
Statement - 2: $|\text{adj } A| = |A|$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is false.
(4) Statement - 1 is false, Statement - 2 is true.

90. **(1) or (2)** $|\text{adj } A| = |A|^{n-1} = |A|$, for $n = 2$

$\text{adj}(A) \cdot \text{adj}(\text{adj}(A)) = |\text{adj}(A)|\, I$

$\Rightarrow \; A \cdot \text{adj}(A) \cdot \text{adj}(\text{adj}(A)) = |\text{adj}(A)|\, A$

$\Rightarrow \; |A| \cdot \text{adj}(\text{adj}(A)) = |A|\, A \quad \Rightarrow \quad \text{adj}(\text{adj}(A)) = A.$

Since 1 does not follow easily from 2, answer could be (2).