Code 0

PART I : CHEMISTRY

PAPER - I

SECTION – I

Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

For the benefit of 11th/12th Studying students, we have (*) marked the questions which are from 11th syllabus. You are advised to solve these questions in 100 minutes.

*1. Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5% respectively, the atomic mass of Fe is

(A) 55.85 (B) 55.95
(C) 55.75 (D) 56.05

Key. **(B)**

Sol. The atomic mass of an element is the average mass number of all its naturally occurring isotopes, the averaging being done on the basis of their respective abundances. Thus

At. Mass of Fe = $\dfrac{5\times 54+90\times 56+5\times 57}{100}=55.95$

Hence (B)

*2. The term that corrects for the attractive forces present in a real gas in the van der Waals equation is

(A) nb (B) $\dfrac{an^2}{V^2}$
(C) $-\dfrac{an^2}{V^2}$ (D) –nb

Key. **(B)**

Sol. Vanderwaals' equation for n moles of a real gas is

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

The pressure correction factor $\dfrac{an^2}{V^2}$ accounts for the loss of pressure due to inward pull. 'a' is vanderwaals' constant for a gas and it is the measure of the strength of the vanderwaals' intermolecular attraction.

Hence $\dfrac{an^2}{V^2}$

3. Among the electrolytes Na_2SO_4, $CaCl_2$, $Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_2 sol is

(A) Na_2SO_4 (B) $CaCl_2$
(C) $Al_2(SO_4)_3$ (D) NH_4Cl

Key. **(C)**

Sol. Sb_2S_3 is a negatively charged colloid.

$\therefore$ cation is responsible for its coagulation and the most effective is $Al_2[SO_4]_3$

1 mol can furnish $2\times Al^{3+}$ ion

i.e. 6 moles of +ve charges which is the maximum in the given choices

Hence (C)

4. The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is

(A) 4.0×10^{-4} (B) 4.0×10^{-5}
(C) 5.0×10^{-4} (D) 4.0×10^{-6}

Key. **(A)**

Sol. $p_{N_2} = 0.8 \times 5 = 4$ atm

From Henry's Law:

$p_{N_2} = K_H \,.\, X_{N_2}$

$$\therefore \quad X_{N_2} = \frac{4}{1\times10^{-5}} = 4\times10^{-5}$$

This means 1 mole of solution will contain 4×10^{-5} mole of N_2 and $1 - 4 \times 10^{-5} \approx 1$ mole water.
Thus, 1 mole of water is association with 4×10^{-5} mole N_2
$\therefore$ 10 moles of water is associated with 4×10^{-5} mole N_2
Hence (A)

5. The reaction of P_4 with X leads selectively to P_4O_6. The X is

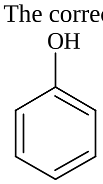
(A) Dry O_2 (B) A mixture of O_2 and N_2
(C) Moist O_2 (D) O_2 in the presence of aqueous NaOH

Key. **(B)**

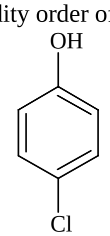
Sol. P_4 gives P_4O_6 when oxygen is in limited supply. Hence a mixture of O_2 and N_2 is most suitable

$P_4 + 3O_2 \longrightarrow P_4O_6$

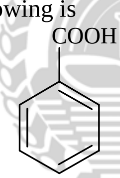
*6. The correct acidity order of the following is



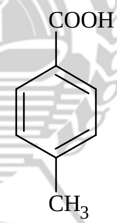
(I)



(II)



(III)

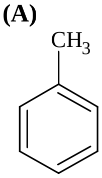


(IV)

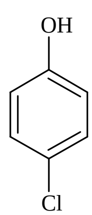
(A) (III) > (IV) > (II) > (I) (B) (IV) > (III) > (I) > (II)
(C) (III) > (II) > (I) > (IV) (D) (II) > (III) > (IV) > (I)

Key. **(A)**

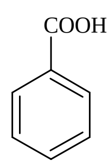
Sol.



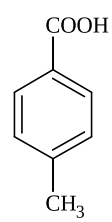
pKa=9.9



pKa = 9.2



pKa = 4.2



pKa = 4.4

Carboxylic acids are more acidic than phenol. EWG exerts acid-strengthening effect while EDG exerts acid-weakening effect. The –I effect of Cl is dominating over its +R effect.
$\therefore$ (A)

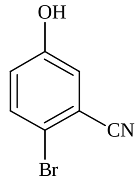
7. Among cellulose, poly(vinyl chloride), nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is

(A) Nylon (B) Poly(vinyl chloride)
(C) Cellulose (D) Natural Rubber

Key. **(D)**

Sol. Nylon and cellulose are Fibres. PVC is a thermoplastic while Natural Rubber is an elastomer. The intermolecular foces of attraction can be graded as
Elastomer < Thermoplastic < Fibres.

*8. The IUPAC name of the following compound is



(A) 4-Bromo-3-cyanophenol
(B) 2-Bromo-5-hydroxybenzonitrile
(C) 2-Cyano-4-hydroxybromobenzene
(D) 6-Bromo-3-hydroxybenzonitrile

Key. **(B)**

Sol. The priority order is:
–CN > Br > –OH
So the compound is
2-bromo 5-hydroxy benzo nitrile

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

9. The correct statement(s) regarding defects in solids is(are)
(A) Frenkel defect is usually favoured by a very small difference in the sizes of cation and anion
(B) Frenkel defect is a disolocation defect
(C) Trapping of an electron in the lattice leads to the formation of F-centre
(D) Schottky defects have no effect on the physical properties of solids

Key. **(B, C)**

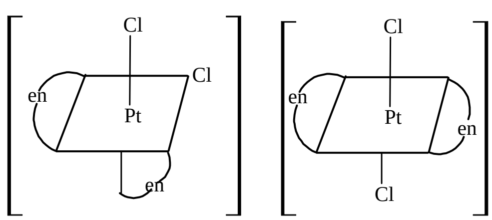
Sol. Frenkel defect is usually favoured by a very large difference in the sizes of cation and anion. In Schottky defect pairs of cations and anions left the crystal lattice as a result density decreases. Hence (B) and (C) are correct.

10. The compound(s) that exhibit(s) geometrical isomerism is(are)
(A) $[Pt(en)Cl_2]$
(B) $[Pt(en)_2]Cl_2$
(C) $[Pt(en)_2Cl_2]Cl_2$
(D) $[Pt(NH_3)_2Cl_2]$

Key. **(C, D)**

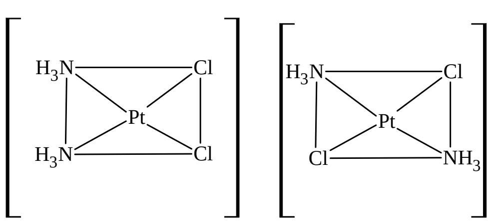
Sol.

(C)



cis trans

(D)



cis trans

*11. The compound(s) formed upon combustion of sodium metal in excess air is(are)

(A) Na_2O_2 (B) Na_2O
(C) NaO_2 (D) NaOH

Key. **(A, B)**

Sol. $2Na + O_2 \longrightarrow Na_2O_2$

$2Na + O_2 \longrightarrow Na_2O$

*12. The correct statement(s) about the compound $H_3C(HO)HC\text{-}CH = CH - CH(OH)CH_3$(X) is(are)
(A) The total number of stereoisomers possible for X is 6
(B) The total number of diastereomers possible for X is 3
(C) If the stereochemistry about the double bond in X is trans, the number of enantiomers possible for X is 4
(D) If the stereochemistry about the double bond in X is cis, the number of enantiomers possible for X is 2

Key. **(A, D)**

SECTION – III
Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

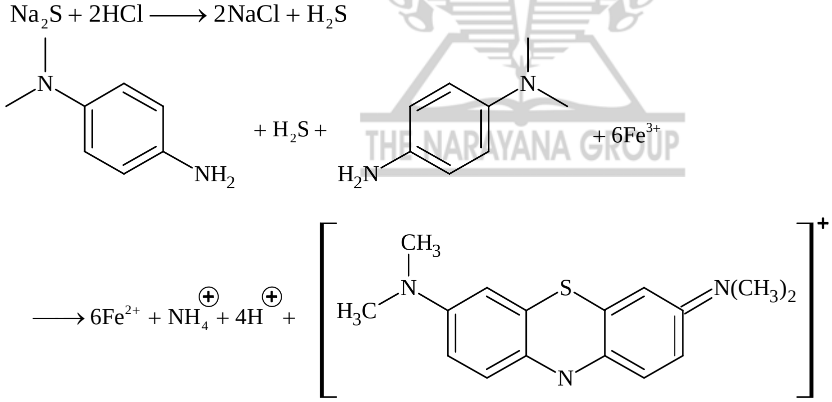
Paragraph for Questions Nos. 13 to 15

p-Amino-N, N-dimethylaniline is added to a strongly acidic solution of X. The resulting solution is treated with a few drops of aqueous solution of Y to yield blue coloration due to the formation of methylene blue. Treatment of the aqueous solution of Y with the reagent potassium hexacyanoferrate(II) leads to the formation of an intense blue precipitate. The precipitate dissolves on excess addition of the reagent. Similarly, treatment of the solution of Y with the solution of potassium hexacyanoferrate (III) leads to a brown coloration due to the formation of Z.

13. The compound X is
(A) $NaNO_3$ (B) NaCl
(C) Na_2SO_4 (D) Na_2S

Key. **(D)**

Sol. $Na_2S + 2HCl \longrightarrow 2NaCl + H_2S$



methylene blue

14. The compound Y is
(A) $MgCl_2$ (B) $FeCl_2$
(C) $FeCl_3$ (D) $ZnCl_2$

Key. **(C)**

Sol. $4Fe^{3+} + 3[Fe(CN)_6]^{4-} \longrightarrow \underset{\text{iron(III) hexacyanoformate}}{\underset{\text{Blue ppt of}}{Fe_4[Fe(CN)_6]_3}}$

The precipitate dissolves in the large excess of the reagent producing an intense blue solution.

15. The compound Z is
(A) $Mg_2[Fe(CN)_6]$ (B) $Fe[Fe(CN)_6]$
(C) $Fe_4[Fe(CN)_6]_3$ (D) $K_2Zn_3[Fe(CN)_6]_2$

Key. **(B)**

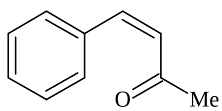
Sol. $Fe^{3+} + [Fe(CN)_6]^{3-} \longrightarrow \underset{\text{Brown colouration}}{Fe[Fe(CN)_6]}$

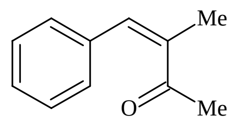
Paragraph for Questions Nos. 16 to 18

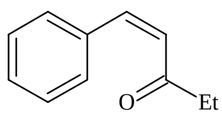
A carbonyl compound P, which gives positive iodoform test, undergoes reaction with MeMgBr followed by dehydration to give an olefin Q. Ozonolysis of Q leads to a dicarbonyl compound R, which undergoes intramolecualr aldol reaction to give predominantly S.

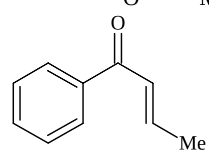
$$P \xrightarrow[\substack{2.\ H^+,\ H_2O \\ 3.\ H_2SO_4,\ \Delta}]{1.\ MeMgBr} Q \xrightarrow[2.\ Zn,\ H_2O]{1.\ O_3} R \xrightarrow[2.\ \Delta]{1.\ OH^-} S$$

16. The structure of the carbonyl compound P is

(A) 

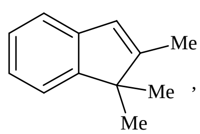
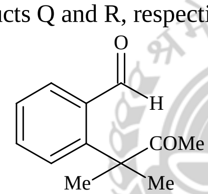
(B) 

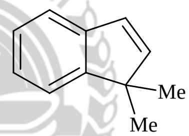
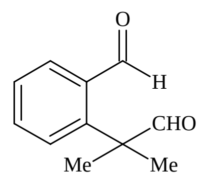
(C) 

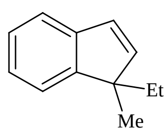
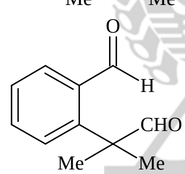
(D) 

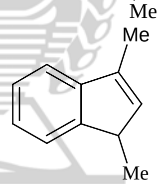
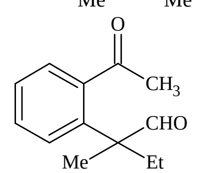
Key. **(B)**

17. The structures of the products Q and R, respectively, are

(A)  , 

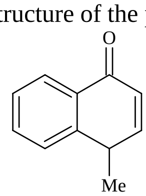
(B)  , 

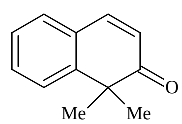
(C)  , 

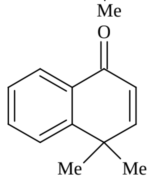
(D)  , 

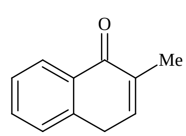
Key. **(A)**

18. The structure of the product S, is

(A) 

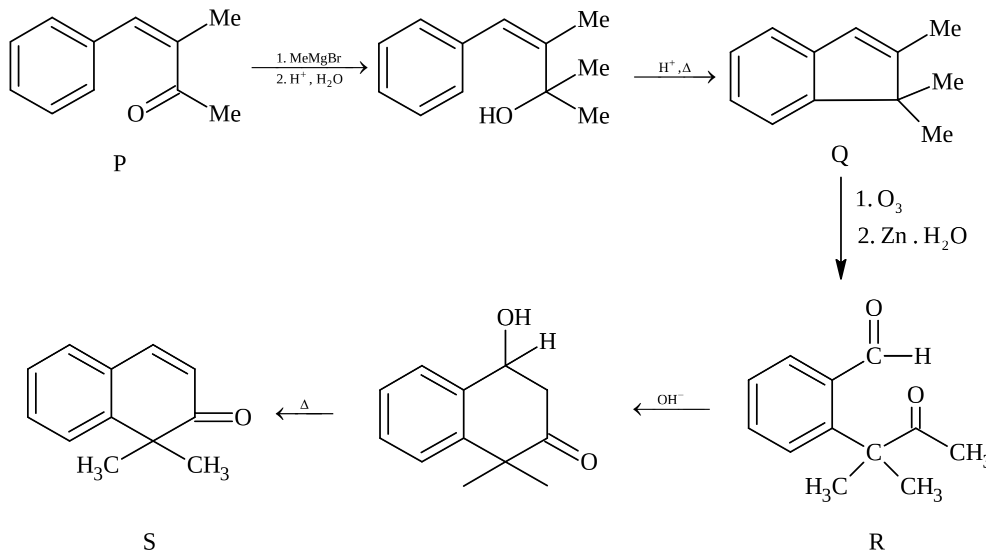
(B) 

(C) 

(D) 

Key. **(B)**

16-18

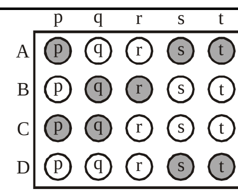


SECTION – IV

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



19. Match each of the diatomic molecules in Column I with its property/properties in Column II

Column I		Column II	
(A)	B_2	(p)	Paramagnetic
(B)	N_2	(q)	Undergoes oxidation
(C)	O_2^-	(r)	Undergoes reduction
(D)	O_2	(s)	Bond order ≥ 2
		(t)	Mixing of ‘s’ and ‘p’ orbital

Key. **(A – p, r, t), (B – s, t), (C – p, q), (D – p, q, s)**

Sol. $B_2\ \sigma_{1s}^2\sigma_{1s}^2\sigma_{2s}^2\sigma_{2s}^2\pi_{2p_x}^1\pi_{2p_y}^1$

$B_2 + e \longrightarrow B_2^-$

$N_2\ \ \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2p_x}^2\pi_{2p_y}^2\sigma_{2p_z}^2$

$O_2\ \ \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\pi_{2p_x}^2\pi_{2p_y}^2\pi_{2p_x}^1\pi_{2p_y}^{*1}$

$O_2^-\sigma_{1s}^2\sigma_{1s}^{+2}\sigma_{2s}^2\sigma_{2s}^{+2}\sigma_{2p_z}^2\pi_{2p_x}^2\pi_{2p_y}^2\pi_{2p_n}^2\pi_{2p_y}^{-1}$

Presence of unpaired electron makes the species paramagnetic. If loss of electron increases the bond order oxidation will be feasible and if the gain of electron increases bond order reduction will be feasible.

$$B.\ O = \frac{1}{2}\left(n_b - n_A\right)$$

$\frac{1}{p}$ to n_2 there is s and p mixing.

20. Match each of the compounds in Column I with its characteristic reaction in Column II.

Column I		**Column II**	
(A)	$CH_3CH_2CH_2CN$	(p)	Reduction with Pd – C/H_2
(B)	$CH_3CH_2OCOCH_3$	(q)	Reduction with $SnCl_2$/HCl
(C)	$CH_3—CH = CH—CH_2OH$	(r)	Development of foul smell on treatment with chloroform and alcoholic KOH
(D)	$CH_3CH_2CH_2CH_2NH_2$	(s)	Reduction with diisobutyl aluminium hydride (DIBAL-H)
		(t)	Alkaline hydrolysis

Key. (A – p, q, s, t), (B – s, t), (C – p), (D – r)

Sol. $R—CN + OH^- \rightarrow RCOO^- + NH_3$

$$R—CN + \xrightarrow{SnCl_2/H^+} R—CH_2NH_2$$

$$CH_5—\overset{\overset{O}{\|}}{C}—OC_2H_5 \xrightarrow{DIBRL—H} CH_3—\overset{\overset{O}{\|}}{C}—H + C_2H_5OH$$

$$CH_3—\overset{\overset{O}{\|}}{C}—OC_2H_5 \xrightarrow{OH^-} CH_3COO^- + C_2H_5OH$$

$$CH_3—CH = CH - CH_2OH \xrightarrow{Pd—C\,|\,H_2} CH_3—CH_2—CH_2—CH_2OH$$

$$RNH_2 \xrightarrow{CHCl_3+KOH} \underset{\text{Foul smell}}{RNC} + KCl + H_2O$$

PART II: MATHEMATICS

SECTION – I
Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

21. Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x - 4y + 3z = 1 is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Key (A)

Sol.: $\overrightarrow{PQ} = \hat{i}\,(-2 - 3\mu) + \hat{j}\,(\mu - 3) + \hat{k}\,(5\mu - 4)$

$\overrightarrow{PQ}$ is parallel to x - 4y + 3z = 1

$\Rightarrow 1(-2 - 3\mu) - 4(\mu - 3) + 3(5\mu - 4) = 0$

$\Rightarrow \mu = \frac{1}{4}$

***22.** Tangents drawn from the point P(1, 8) to the circle
$x^2 + y^2 - 6x - 4y - 11 = 0$
touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

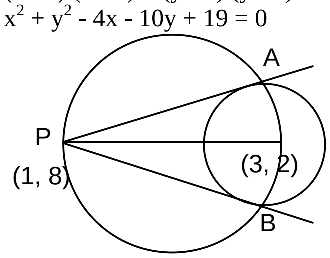
(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Key (B)

Sol.: $(x - 1)(x - 3) + (y - 8)(y - 2) = 0$

$x^2 + y^2 - 4x - 10y + 19 = 0$



23. Let f be a non-negative function defined on the interval [0, 1]. If

$\int_0^x \sqrt{1 - (f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$, and f(0) = 0, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Key. (C)

Sol.: $\sqrt{1 - (f'(x))^2} = f(x)$,

$1 - (f'(x))^2 = (f(x))^2$

$(f'(x))^2 = 1 - (f(x))^2$

Let y = f(x)

$\frac{dy}{dx} = f'(x)$

$\left(\frac{dy}{dx}\right) = \pm\sqrt{1-y^2}$

$\int \frac{dy}{\pm\sqrt{1-y^2}} = \int dx$

$\sin^{-1}y = \pm(x) + c$

$f(0) = 0 \Rightarrow\ c = 0$

$\Rightarrow f(x) = \sin x$

$(\therefore\ f(x)$ is non-negative)

as $\sin x < x\ \forall\ x > 0$

$\Rightarrow f\left(\frac{1}{2}\right) = \sin\ \frac{1}{2} < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

***24.** Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation

$z\bar{z}^3 + \bar{z}z^3 = 350$ is

(A) 48 (B) 32

(C) 40 (D) 80

Key (A)

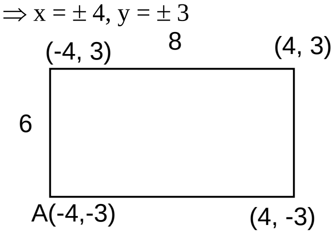
Sol.: $Z\bar{Z}^3 + \bar{Z}Z^3 = 350$

Let $Z = x + iy$ $(x, y \in Z)$

$(x^2 + y^2)(x^2 - y^2) = 175$

$\Rightarrow x^2 + y^2 = 25\ |x^2 - y^2 = 7$

$\Rightarrow x = \pm 4, y = \pm 3$



$\Rightarrow$ area = 48 sq. units.

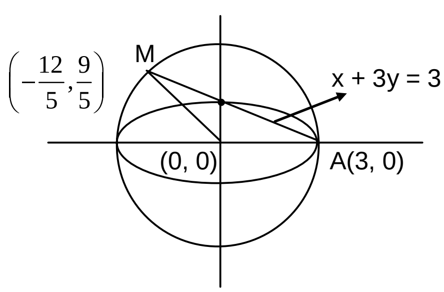
***25.** The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$

(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Key (D)

Sol.:



Area of Δle OAM = $\frac{1}{2}\left(\frac{27}{5}\right) = \frac{27}{10}$

26. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar
(B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
(C) $\vec{b}, \vec{d}$ are non-parallel
(D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

Key (C)

Sol.: $\bar{a}\times\bar{b} = |\bar{a}||\bar{b}|\sin\alpha\,\hat{n}$, $\alpha \in [0, \pi] = \sin\alpha\ \hat{n}_1$

$\bar{c}\times\bar{d} = \sin\beta\ \hat{n}_2$, $\beta \in [0, \pi]$

$(\bar{a}\times\bar{b}).(\bar{c}\times\bar{d}) = 1$

$\Rightarrow \sin\alpha\,.\,\sin\beta\ (\hat{n}_1.\hat{n}_2) = 1$ where θ is the angle between $\vec{n}_1 \,\&\, \vec{n}_2$

$\Rightarrow \sin\alpha\sin\beta\cos\theta = 1$

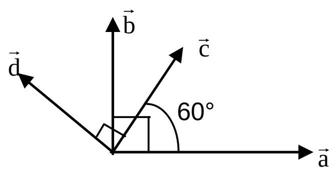
$\Rightarrow \sin\alpha = 1, \sin\beta = 1$ and $\cos\theta = 1$

$\Rightarrow \alpha = \pi/2, \beta = \pi/2$ and $\theta = 0$

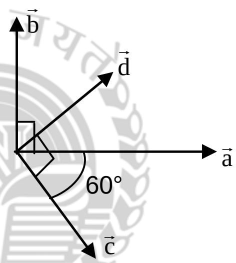
Now, $\bar{a}.\bar{c} = \frac{1}{2} \Rightarrow \cos\gamma = 1/2 \Rightarrow \gamma = \pi/3$

As $\bar{a}\times\bar{b} \parallel \bar{c}\times\bar{d}$, $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are coplanar

There are two possibilities as shown



or



So option (C) is correct

***27.** Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}\ (z^{2m-1})$ at $\theta = 2°$ is

(A) $\frac{1}{\sin 2°}$
(B) $\frac{1}{3\sin 2°}$
(C) $\frac{1}{2\sin 2°}$
(D) $\frac{1}{4\sin 2°}$

Key (D)

Sol. $\sum_{m=1}^{15} \text{Im}\ (Z^{2m-1}) = \text{Im} \sum_{m=1}^{15} Z^{2m-1}$

$= \text{Im}\left[\frac{Z(1-(Z^2)^{15})}{1-Z^2}\right] = \text{Im}\left[\frac{1-Z^{30}}{\frac{1}{Z}-Z}\right]$

$= \text{Im}\left[\frac{1-\cos 30\theta - i\sin 30\theta}{-2i\sin\theta}\right]$

$= \frac{1-\cos 30\theta}{2\sin\theta} = \frac{1-\cos 60°}{2\sin 2°} = \frac{1}{4\sin 2°}$

***28.** The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55
(B) 66
(C) 77
(D) 88

Key (C)

Sol.: **Case I:** digits used 1, 1, 1, 1, 1, 3, 2

Number of integers formed = $\frac{7!}{5!}$ = 42

Case II: digits used : 1, 1, 1, 1, 2, 2, 2

Number of integers formed = $\frac{7!}{3!\,4!}$ = 35

Total number of integers formed = 77.

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

29. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(A) e - 1

(B) $\int_1^e \ln\,(e + 1 - y)\,dy$ *s*

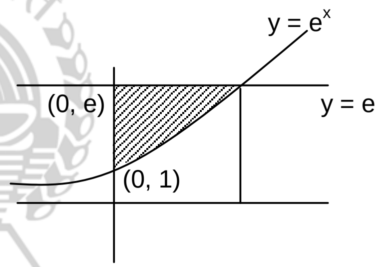
(C) e - $\int_0^1 e^x\,dx$

(D) $\int_1^e \ln y\,dy$ *s*

Key: (B, C, D)

Sol.: Area = e - $\int_0^1 e^x\,dx$

= $\int_1^e \ln y\,dy$



30. Let $L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, a > 0. If L is finite, then

(A) a = 2

(B) a = 1

(C) L = $\frac{1}{64}$

(D) L = $\frac{1}{32}$

Key (A, C)

Sol.: $L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$ **a > 0**

$$\lim_{x\to 0} \frac{a - (a^2 - x^2)^{1/2} - \frac{x^2}{4}}{x^4}$$

$$\lim_{x\to 0} \frac{a - a\left(1 - \left(\frac{x}{a}\right)^2\right)^{1/2} - \frac{x^2}{4}}{x^4}$$

$$\lim_{x\to 0} \frac{a - a\left(1 - \frac{1}{2}, \frac{x^2}{a^2} - \frac{1}{2}.\frac{1}{2}.\frac{x^4}{2a^4}\right) - \frac{x^2}{4}}{x^4}$$

$$\lim_{x\to 0}\frac{\frac{1}{2}\frac{x^2}{a}+\frac{1}{8}.\frac{x^4}{a^4}-\frac{x^2}{4}}{x^4}$$

$$\frac{x^2\left(\frac{1}{2a}-\frac{1}{4}\right)+\frac{1}{8}.\frac{x^4}{a^3}}{x^4}$$

If $\frac{1}{2a}-\frac{1}{4}=0$ a = 2

if a = 2

L = $\frac{1}{8}.\frac{1}{8}=\frac{1}{64}$

***31.** In a triangle ABC with fixed base BC, the vertex A moves such that cosB + cosC = 4 $\sin^2\frac{A}{2}$.

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then

(A) b + c = 4a
(B) b + c = 2a
(C) locus of point A is an ellipse
(D) locus of point A is a pair of straight lines

Key (B, C)

Sol.: $\cos B + \cos C = 4\sin^2\frac{A}{2}$

$\Rightarrow 2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right) = 4\sin^2 A/2$

$\Rightarrow \cos\left(\frac{B-C}{2}\right)=2\sin(A/2)$

$\Rightarrow 2\cos\frac{A}{2}\cos\frac{B-C}{2}=4\sin\frac{A}{2}\cos\frac{A}{2}$

$\Rightarrow$ sinB + sinC = 2sinA

$\Rightarrow$ b + c = 2a

***32.** If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then

(A) $\tan^2 x = \frac{3}{2}$
(B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$,
(C) $\tan^2 x = \frac{1}{3}$
(D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

Key (A, B)

Sol.: $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}=\frac{(\sin^2 x+\cos^2 x)^2}{5}$

$\Rightarrow 9\sin^4 x + 4\cos^4 x - 12\sin^2 x\cos^2 x = 0$

$\Rightarrow (3\sin^2 x - 2\cos^2 x)^2 = 0$

$\Rightarrow \frac{\sin^2 x}{2}=\frac{\cos^2 x}{3}=\frac{1}{5}$

$\Rightarrow \sin^2 x = \frac{2}{5}, \cos^2 x = \frac{3}{5}$

$\Rightarrow \tan^2 x = 2/3$

and $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27} = \frac{2}{5^4} + \frac{3}{5^4} = \frac{1}{5^3}=\frac{1}{125}$

Alternate:

As we know that

$\frac{x^2}{a}+\frac{y^2}{b}\geq\frac{(x+y)^2}{a+b}$ for x, y $\in R$ and a, b $\in R^+$

Also equality holds when $\frac{x}{a}=\frac{y}{b}$

$\Rightarrow \frac{(\sin^2 x)^2}{2}+\frac{(\cos^2 x)^2}{3}\geq\frac{(\sin^2 x+\cos^2 x)^2}{2+3}=\frac{1}{5}$

$\Rightarrow \frac{\sin^2 x}{2}=\frac{\cos^2 x}{3}=\frac{1}{5}$

$\Rightarrow \sin^2 x = \frac{2}{5}, \cos^2 x = \frac{3}{5}$

$\Rightarrow \tan^2 x = 2/3$ and $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27} = \frac{2}{5^4} + \frac{3}{5^4} = \frac{1}{5^3}=\frac{1}{125}$

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 33 to 35

Let $\mathcal{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

33. The number of matrices in $\mathcal{A}$ is

(A) 12 (B) 6

(C) 9 (D) 3

Key (A)

Sol.: (A)

There are two cases

Case I : diagonal elements 1, 0, 0

Number of symmetric matrices

= Arrangement of main diagonal elements × Arrangement of remaining elements

$=\frac{3!}{2!}\times\frac{3!}{2!}$

= 9

Case II : diagonal elements 1, 1, 1

No. of symmetric matrices $=\frac{3!}{2!}=3$

Total symmetric matrices are 9 + 3 = 12

34. The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

Key (B)

Sol.: Consider symmetric matrix.

$$A=\begin{bmatrix}a & h & g\\h & b & f\\g & f & c\end{bmatrix}$$

$|A| = abc + 2fgh - af^2 - bg^2 - ch^2$

Case I : when a = b = c = 1 then out f, g, h two are '0' and remaining '1'

⇒ $|A| = 0$

⇒ there are three such matrices.

Case II : when either of a, b or c = 1 and other two are zero

then $abc = 0, fgh = 0 \Rightarrow |A| = -af^2$ or $-bg^2$ or $-ch^2$

when a = 1, and f = 0

⇒ $|A| = 0$

⇒ there are three such matrices

total number of matrices such that $|A| = 0$

$= 3 + 3 = 6$

⇒ Total number of matrices such that $|A| \neq 0$ is $12 - 6 = 6$

35. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2

(C) 2 (D) 1

Key (B)

Sol.: $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$ax + hy + gz = 1$

$hx + by + fz = 0$

$gx + fy + cz = 0$

when $a = b = c = 1$

then system will be inconsistent when h = 1 or g = 1.

When $a = b = 0, c = 1$

Then system will be inconsistent when h = 0

Hence more than 2 matrices.

Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

36. The probability that X = 3 equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$

(C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Key (A)

Sol.: $P(X = 3) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{25}{216}$

37. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$

(C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Key (B)

Sol.: $P(X \geq 3) = \frac{5}{6}\times\frac{5}{6} = \frac{25}{36}$

38. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$

(C) $\frac{5}{36}$ (D) $\frac{25}{36}$

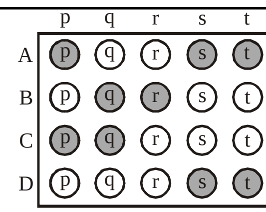
Key (D)

Sol.: $P\left(\frac{X \geq 6}{X > 3}\right) = \frac{P(x \geq 6)}{P(x > 3)} = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$

SECTION – IV

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*
If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



39. Match the statements/expressions in Column I with the open intervals in Column II.

	Column I		Column II
(A)	Interval contained in the domain of definition of non-zero solutions of the differential equation $(x - 3)^2 y' + y = 0$	(p)	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B)	Interval containing the value of the interval $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q)	$\left(0, \frac{\pi}{2}\right)$
		(r)	$\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(C)	Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(s)	$\left(0, \frac{\pi}{8}\right)$
(D)	Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s)	$(-\pi, \pi)$

Key **(A-p, q, s), (B-p, t), (C-p, q, r, t) (D-s)**

Sol.: (A) $\frac{dy}{y} + \frac{dx}{(x-3)^2} = 0$

$\ln|y| - \frac{1}{x-3} = \ln c$

$y = ce^{\frac{1}{x-3}}$, domain $R - \{3\}$

p,q,s

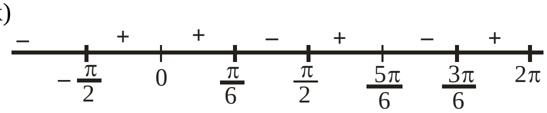
(B) Let I = $\int_1^5 (x+2)(x+1)x(x-2)(x-1)dx$

$\Rightarrow$ I = $\int_1^5 (6-x-1)(6-x-2)(6-x-3)(6-x-4)(6-x-5)dx$

$= -\int_1^5 (x+2)(x+1)x(x-2)(x-1)dx$

$\Rightarrow I = -I \quad \Rightarrow I = 0$
p, s, t

(C) $f(x) = \cos^2 x + \sin x$
$f(x) = \cos x(1 - 2\sin x)$



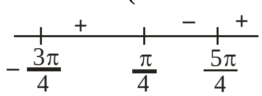
points of max. are

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

p, q, r, t

(D) $f(x) = \tan^{-1}(\sin x + \cos x)$

$$f'(x) = \frac{1(\cos x - \sin x)}{1 + (\sin x + \cos x)^2}$$



***40.** Match the conics in Column I with the statements/expressions in Column II.

(A)	Circle	(p)	The locus of the point (h, k) for which the line hx + ky = 1 touches the circle $x^2 + y^2 = 4$
(B)	Parabola	(q)	Points z in the complex plane satisfying $\lvert z + 2\rvert \lvert z - 2\rvert = \pm 3$
		(r)	Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(C)	Ellipse	(s)	The eccentricity of the conic lies int he interval $1 \le x < \infty$
(D)	Hyperbola	(s)	Points z in the complex plane satisfying Re $(z + 1)^2 = \lvert z\rvert^2 + 1$

Key **(A-p), (B-s, t) (C-r) (D-q, s)**

Sol.: (p) hx + ky = 1 touches $x^2 + y^2 = 4$

$$\Rightarrow \quad \left|\frac{-1}{\sqrt{h^2 + k^2}}\right| = 2 \quad \Rightarrow \quad h^2 + k^2 = \frac{1}{4} \text{ (circle)}$$

(q) $||z + 2| - |z - 2|| = 3$
$2ae = 4, \quad 2a = 3$

$$e = \frac{4}{3} > 1 \text{ i.e.} \qquad \text{(hyperbola)}$$

(r) $$x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), \quad y = \frac{2t}{1+t^2} \text{ put } t = \tan\theta$$

$$\Rightarrow x = \sqrt{3}\cos 2\theta$$

$y = \sin 2\theta$

$$\Rightarrow \quad \left(\frac{x}{\sqrt{3}}\right)^2 + y^2 = 1 \qquad \text{(ellipse)}$$

(s) Clearly parabola or hyperbola
(t) Let z = x + iy
$\text{Re}(z + 1)^2 = (x + 1)^2 - y^2 = x^2 + y^2 + 1$
$y^2 = x$ (parabola)

PART III: PHYSICS

Useful Data:
Planck's constant $h = 4.1 \times 10^{-14}$ eV.s
Velocity of light $c = 3 \times 10^{8}$ m/s.

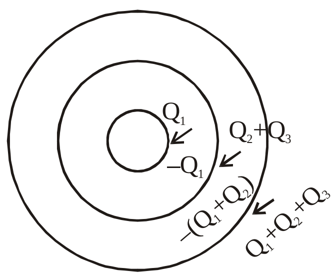
SECTION – I
Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

41. Three concentric metallic spherical shells of radii R, 2R, 3R, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) 1 : 2 : 3 (B) 1 : 3 : 5
(C) 1 : 4 : 9 (D) 1 : 8 : 18.

Key. (B)
Sol.



It is given that

$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi (2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi (3R)^2}$$

i.e., $\frac{Q_1}{1} = \frac{Q_1 + Q_2}{4} = \frac{Q_1 + Q_2 + Q_3}{9}$

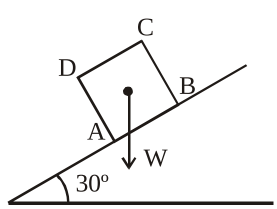
or, $\frac{Q_1}{1} = \frac{Q_2}{3} = \frac{Q_3}{5}$

So, option (B) is correct.

42. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased form 0º. Then

(A) at θ = 30º, the block will start sliding down the plane
(B) the block will remain at rest on the plane up to certain θ and then it will topple
(C) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles
(D) at θ = 60º, the block will start sliding down the plane and further increasing θ, it will topple at certain θ.

Key. (B)
Sol.



At θ = 30º, the weight W of the block passes through the base AB, and hence the block will not topple; since

$$\tan 30^\circ = \frac{1}{\sqrt{3}} < \sqrt{3} \text{ it will not slide.}$$

However, as θ is increased the block will topple when

$\theta + \tan^{-1} 1.5 = 90^\circ$

or, $\theta = 90^\circ - \tan^{-1} 1.5 = \tan^{-1} \frac{2}{3} < 60^\circ$

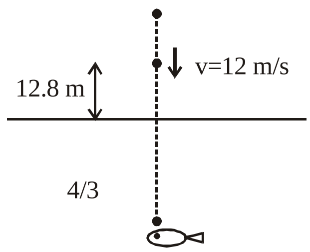
So, option (B) is correct.

43. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of the ball as [Take g = 10 m/s^2.]

(A) 9 m/s (B) 12 m/s
(C) 16 m/s (D) 21.23 m/s.

Key. (C)

Sol.



$v = \sqrt{2 \times 10 \times (20 - 12.8)} = 12\ \text{m/s}$

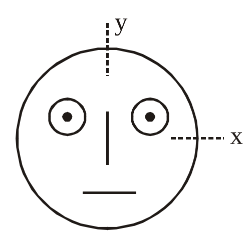
Due to refraction at the water surface,

$\frac{v}{\mu_2} = \frac{u}{\mu_1}$

or $\frac{dv}{dt} = \frac{\mu_2}{\mu_1} \cdot \frac{dv}{dt}$

$\left|\frac{dv}{dt}\right| = \frac{4}{3} \times 12 = 16\ \text{m/s}$. So, option (C) is correct.

*44. Look at the drawing given in the figure which has been drawn with ink of uniform line thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centers of the different parts are outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y–coordinate of the center of mass of the ink in this drawing is

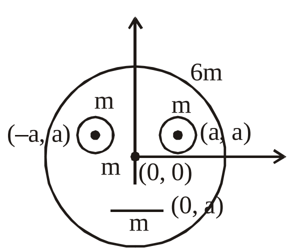


(A) $\frac{a}{10}$ (B) $\frac{a}{8}$

(C) $\frac{a}{12}$ (D) $\frac{a}{3}$.

Key. (A)

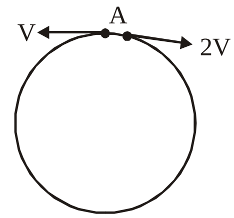
Sol.



$y_{cm} = \frac{7m \times 0 + 2m \times a + m(-a)}{10m} = \frac{ma}{10m} = \frac{a}{10}$

So, option (A) is correct.

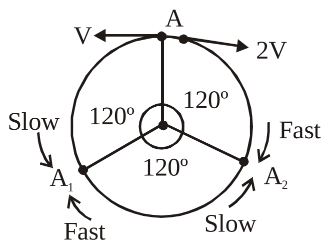
*45. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A ?



(A) 4 (B) 3
(C) 2 (D) 1.

Key. (C)

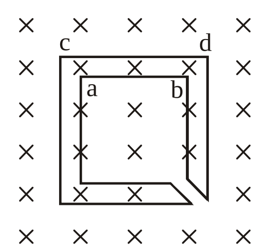
Sol.



The first collision takes place at A_1 as the particle moving in anticlockwise sense covers half the distance covered by other. They exchange velocities and the second collision takes place at A_2. The third collision takes place at A.

So, option (C) is correct.

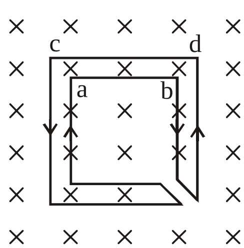
46. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time I_1 and I_2 are the currents in the segments ab and cd. Then,



(A) $I_1 > I_2$
(B) $I_1 < I_2$
(C) I_1 is in the direction ba and I_2 is in the direction cd
(D) I_1 is in the direction ab and I_2 is in the direction dc.

Key. (D)

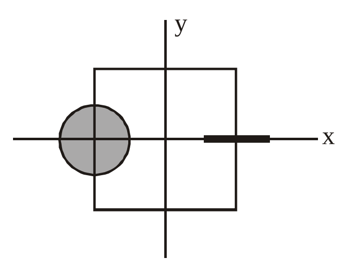
Sol.



Since the field is increasing, the flux region is increasing; the induced current (by Lenz's law) should flow in an anticlockwise sense around the boundary of the shaded region.

So, option (D) is correct.

47. A disk of radius a/4 having a uniformly distributed charges 6C is placed in the x–y plane with its center at (–a/2, 0, 0). A rod of length a carrying a uniformly distributed charge 8C on the x–axis from x = a/4 to x =5a/4. Two point charges –7C and 3C are placed at (a/4, –a/4, 0) and (–3a/4, 3a/4, 0), respectively. Consider a cubical surface formed by six surfaces x = ± a/2, y = ± a/2, z = ± a/2. The electric flux through this cubical surface is

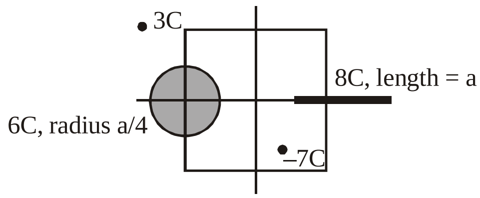


(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$

(C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$.

Key. (A)

Sol.

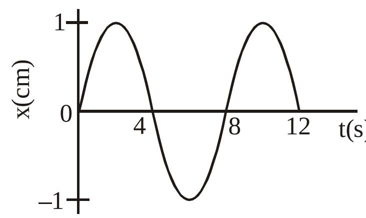


Net charge $= \dfrac{-7+3+2}{\varepsilon_0}$

$= \dfrac{-2C}{\varepsilon_0}$

So, option (A) is correct.

*48. The x–t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is

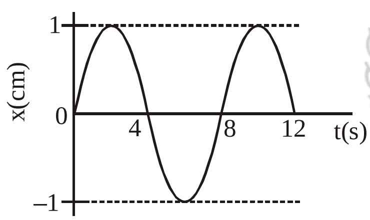


(A) $\dfrac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2$ (B) $\dfrac{\pi^2}{32}\ \text{cm/s}^2$

(C) $\dfrac{\pi^2}{32}\ \text{cm/s}^2$ (D) $-\dfrac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2$.

Key. (D)

Sol.



$x = 1\sin\left(\dfrac{2\pi}{8}t\right)$ cm

Acceleration acceleration $\left|= \dfrac{d^2x}{dt^2}\right|_{t=4/3} = -\left(\dfrac{\pi}{4}\right)^2 \sin\left(\dfrac{\pi t}{4}\right)\Bigg|_{t=4/3}$

$= -\dfrac{\pi^2}{16}\times\dfrac{\sqrt{3}}{2} = -\dfrac{\sqrt{3}\pi^2}{32}\ \text{cm/s}^2$

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

*49. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time.

Key. (A, C)

Sol. $\sum \vec{F}_{ext} = \vec{0} \Rightarrow \vec{P} = \text{constant}$

So, option (A) is correct.

50. A student performed the experiment of determination of focal length of a concave mirror by u–v method using an optical bench of length 1.5 meter. The length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)

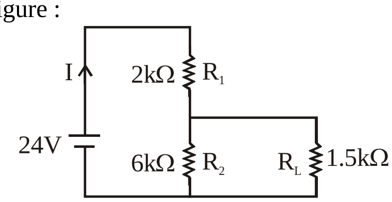
(A) (42, 56) (B) (48, 48)
(C) (66, 33) (D) (78, 39).

Key. (C, D)

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

the data set (66, 33) does not satisfy the mirror equation.
So, options (C) and (D) are correct.

51. For the circuit shown in the figure :



(A) the current I through the battery is 7.5 mA
(B) the potential difference across R_L is 18 V
(C) ratio of powers dissipated in R_1 and R_2 is 3
(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9.

Key. (A, D)

Sol. $R_{eq} = 3.2\,k\Omega$

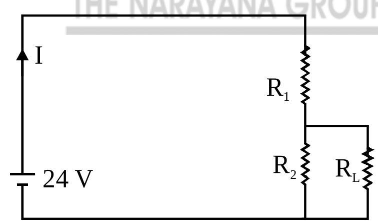
$I = \frac{24}{3.2} = 7.5\,mA$ & $I_{L_o} = 6\,mA$

After interchanging R_1 with R_2

$R_{eq} = \frac{48}{7}\Omega$

So, I = 3.5 mA

$I_{L_1} = 2\,mA$



So, $\frac{P_{L_o}}{P_{L_1}} = \frac{36}{4} = 9$

*52. C_V and C_P denote the molar specific heat capacitors of a gas at constant volume and constant pressure, respectively. Then

(A) $C_P - C_V$ is larger for a diatomic ideal gas then for a monoatomic ideal gas
(B) $C_P + C_V$ is larger for a diatomic ideal gas then for a monoatonic ideal gas
(C) C_P / C_V is larger for a diatomic ideal gas then for a monoatonic ideal gas
(D) $C_P . C_V$ is larger for a diatomic ideal gas then for a monoatonic ideal gas.

Key. (B, D)

Sol. C_P and C_V for diatomic is greater than monoatomic.
So, $C_P + C_v$, $C_P . C_V$ is greater for diatomic ideal gas.

SECTION – III
Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 53 to 55

Scientists are working hard to develop unclear fusion reactor. Nuclei of heavy hydrogen, ${}^{2}_{1}H$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D–D reaction is ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^{2}_{1}H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following : Boltzmann constant k = 8.6×10^{-5} eV/K; $\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

53. In the core of nuclear fusion reactor, the gas becomes plasma because of
(A) strong nuclear force acting between the deuterons
(B) Coulomb force acting between the deuterons
(C) Coulomb force acting between the deuteron–electron pairs
(D) the high temperature maintained inside the reactor core.

Key. (D)

Sol. Plasma state is achieved at high temperatures.

54. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range
(A) 1.0×10^9 K < T < 2.0×10^9 K
(B) 2.0×10^9 K < T < 3.0×10^9 K
(C) 3.0×10^9 K < T < 4.0×10^9 K
(D) 4.0×10^9 K < T < 5.0×10^9 K.

Key. (A)

Sol. (A)

$$3kt = \frac{e^2}{4\pi\varepsilon_0 d}$$

$$T = \frac{e^2}{3 \times 4\pi\varepsilon_0 kd} = \frac{1.44 \times 10^{-9}\,eV_m}{3 \times 8.6 \times 10^{-5}\,eV/K \times 4 \times 10^{-15}\,m} = \frac{120}{86} \times 10^9 \approx 1.4 \times 10^9$$

55. Results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is most promising based on Lawson criterion ?
(A) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s
(B) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s
(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s
(D) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s.

Key. (B)

Sol. (B)
Lawson no. = nt_o Out of given option nt_o is greater for option (B).

Paragraph for Questions Nos. 56 to 58

When a particle is restricted to move along x–axis between x = 0 and x = a, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends x = 0 and x = a. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de–Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number n taking values 1, 2, 3, … (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line x = 0 to x = a. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

56. The allowed energy for the particle for a particular value of n is proportional to
(A) a^{-2} (B) $a^{-3/2}$
(C) a^{-1} (D) a^2.

Key. (A)

Sol. $$a = n\left(\frac{\lambda}{2}\right) \Rightarrow \lambda = \frac{2a}{n}$$
$$p = \frac{h}{\lambda} = \frac{n \cdot h}{2a}$$
$$E = \frac{n^2h^2}{4a^2 2m} = \frac{n^2h^2}{8ma^2}$$
$$\Rightarrow \quad E \propto a^{-2}$$

57. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and a = 6.6 nm, the energy of the particle in its ground state is closest to
(A) 0.8 meV (B) 8 meV
(C) 80 meV (D) 800 meV.

Key. (B)

Sol. In ground state n = 1
$$E = \frac{h^2}{8ma^2} = \frac{\left(6.6\times10^{-34}\right)^2}{8\times10^{-30}\times\left(6.6\times10^{-9}\right)^2} J$$
$$= \frac{1}{8}\times\frac{10^{-68}}{10^{-48}\times1.6\times10^{-19}} = \frac{10^{-1}}{8\times1.6} eV = \frac{100}{8\times1.6} meV = 8 meV$$

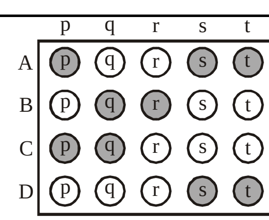
58. The speed of the particle, that can take discrete values, is proportional to
(A) $n^{-3/2}$ (B) n^{-1}
(C) $n^{1/2}$ (D) n.

Key. (D)

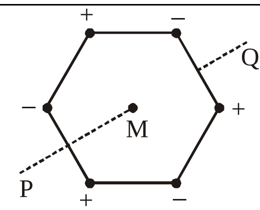
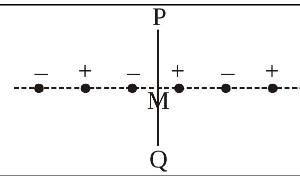
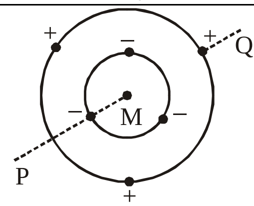
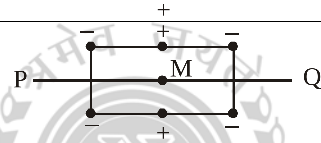
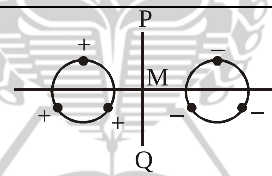
Sol. $$\frac{1}{2}mv^2 = \frac{n^2h^2}{8ma^2} \quad \Rightarrow v \propto n$$

SECTION – IV
Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*
If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



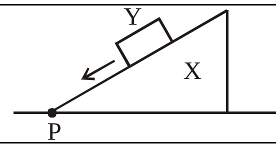
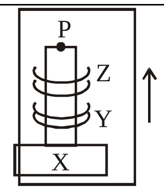
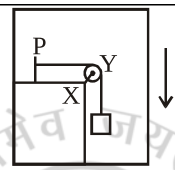
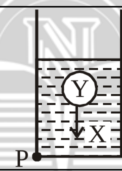
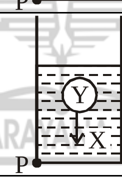
59. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

<table>
<tr><th colspan="2">Column I</th><th colspan="3">Column II</th></tr>
<tr><td>(A)</td><td>E = 0</td><td>(p)</td><td>
</td><td>Charges are at the corners of a regular hexagon. M is at the center of the hexagon. PQ is perpendicular to the plane of the hexagon.</td></tr>
<tr><td>(B)</td><td>$V \neq 0$</td><td>(q)</td><td>
</td><td>Charge are on a line perpendicular to PQ at equal intervals. M is the mid–point between the two innermost charges.</td></tr>
<tr><td>(C)</td><td>B = 0</td><td>(r)</td><td>
</td><td>Charges are placed on two coplanar insulating rings at equal intervals. M is the common center of the rings. PQ is perpendicular to the plane of the rings.</td></tr>
<tr><td>(D)</td><td>$\mu \neq 0$</td><td>(s)</td><td>
</td><td>Charges are placed at the corners of a rectangle of sides a and 2a and at the mid points of the longer sides. M is at the center of the rectangle. PQ is parallel to the longer sides.</td></tr>
<tr><td></td><td></td><td>(t)</td><td>
</td><td>Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid–point between the centers of the rings. PQ is perpendicular to the line joining the centers and coplanar to the rings.</td></tr>
</table>

Key. **(A –p, r, s), (B –r, s), (C –p, q, t), (D – r, s)**

Sol. In (q) and (t) at point M electric field is not zero but electric potential is zero. When system is rotated about line PQ then equivalent current is zero.
In part (r) field at point M is zero but potential is not zero.
In part (s) at point M both field and potential are non zero.

*60. Column II shows five system in which two objects the labelled as X and Y. Also in each case a point P is shown. Column I gives some statements about X and / or Y. Match these statements to the appropriate system (s) from Column II.

Column I		Column II		
(A)	The force exerted by X on Y has a magnitude Mg.	(p)		Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.
(B)	The gravitational potential energy of X is continuously increasing.	(q)		Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.
(C)	Mechanical energy of the system X + Y is continuously decreasing.	(r)		A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in lift that is going down with a constant velocity.
(D)	The torque of the weight of Y about point P is zero.	(s)		A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.
		(t)		A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Key. **(A – p, t), (B –q, s, t), (C –p, r, t), (D –q)**

Sol. When a ball moves in a liquid in downward direction then centre of mass of the liquid moves in upward direction.

PART – I: CHEMISTRY

PAPER - II

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

For the benefit of 11th/12th Studying students, we have (*) marked the questions which are from 11th syllabus. You are advised to solve these questions in 100 minutes.

1. For a first order reaction A $\rightarrow$ P, the temperature (T) dependent rate constant (k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are

(A) $1.0 \times 10^6 s^{-1}$ and 9.2 kJ mol^{-1} (B) 6.0 s^{-1} and 16.6 kJ mol^{-1}
(C) $1.0 \times 10^6\ s^{-1}$ and 16.6 kJ mol^{-1} (D) $1.0 \times 10^6\ s^{-1}$ and 38.3 kJ mol^{-1}

Key. **(D)**

Sol. $k = Ae^{-Ea/RT}$

$\log k = \log A - \frac{E_a}{2.303RT}$

Log A = 6, A = $10^6 s^{-1}$

$-\frac{E_a}{2.303 \times 8.3 \times T} = -\frac{2000}{T}$

$E_a = 2000 \times 2.303 \times 8.3$ J

= 38.3 kJ

2. The spin only magnetic moment value (in Bohr magneton units) of $Cr(CO)_6$ is

(A) 0 (B) 2.84
(C) 4.90 (D) 5.92

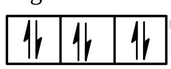
Key. **(A)**

Sol. $Cr(CO)_6$

Cr(zero)

Atomic configuration : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1\ 3d^5$

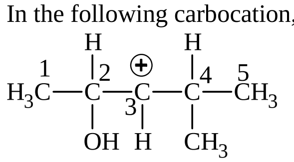
CO is a strong field ligand

$\therefore$ Configuration  t_{2g}

No. of unpaired electron = 0

$\therefore$ magnetic moment = 0

*3. In the following carbocation, H/CH_3 that is most likely to migrate to the positively charged carbon is



(A) CH_3 at C-4 (B) H at C-4
(C) CH_3 at C-2 (D) H at C-2

Key. **(D)**

Sol. Hydride shift from C-2 will yield resonance stabilized 2°-carbocation giving thereby ketonic product after deprotonation.

*4. The correct stability order of the following resonance structures is

$\underset{(I)}{H_2C = \overset{+}{N} = \overset{-}{N}}$ $\underset{(II)}{H_2\overset{+}{C} - N = \overset{-}{N}}$ $\underset{(III)}{H_2\overset{-}{C} - \overset{+}{N} \equiv N}$ $\underset{(IV)}{H_2\overset{-}{C} - N = \overset{+}{N}}$

(A) (I) > (II) > (IV) > (III) (B) (I) > (III) > (II) > (IV)
(C) (II) > (I) > (III) > (IV) (D) (III) > (I) > (IV) > (II)

Key. **(B)**

Sol. In I and III all the atoms fulfil the octet requirement.
Between II and IV, structure II has negative charge on nitrogen atom. Whereas in IV –ve charge occurs at carbon which is less electronegative.

SECTION – II
Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

5. For the reduction of NO_3^- ioin in an aqueous solution E is +0.96 V. Values of E° for some metal ions are given below

V^{2+} (aq) + $2e^- \rightarrow$ V	$E^0 = -1.19V$
Fe^{3+} (aq) + $3e^- \rightarrow$ Fe	$E^0 = -0.04V$
Au^{3+} (aq) + $3e^- \rightarrow$ Au	$E^0 = +1.40$ V
Hg^{2+} (aq) + $2e^- \rightarrow$ Hg	$E^0 = +0.86V$

The pair(s) of metals that is(are) oxidized by NO_3^- in aqueous solution is(are)
(A) V and Hg (B) Hg and Fe
(C) Fe and Au (D) Fe and V

Key. **(A, B, D)**

Sol. NO_3^- ion will oxidise all those metal ions whose $E^o_{reduction}$ is less than 0.96V

*6. Among the following, the state function(s) is(are)
(A) Internal energy (B) Irreversible expansion work
(c) Reversible expansion work (d) Molar enthalpy

Key. **(A, D)**

Sol. ΔE and ΔH are path independent and are definite quantities in a given change of states. Hence, E and H are state function.

***7.** In the reaction
$2X + B_2H_6 \rightarrow [BH_2(X)_2]^+ [BH_4]^-$
The amine(s) X is(are)
(A) NH_3 (B) CH_3NH_2
(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

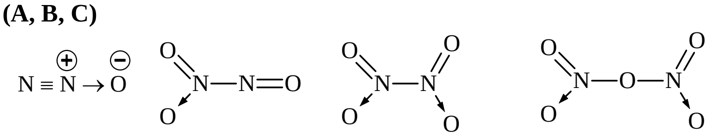
Key. **(A, B, C)**

Sol. 3°-Amine form some different kind of complex with diborance

8. The nitrogen oxide(s) that contain(s) N-N bond(s) is(are)
(A) N_2O (B) N_2O_3
(C) N_2O_4 (D) N_2O_5

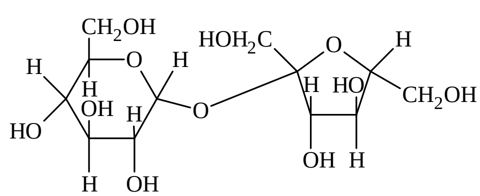
Key. **(A, B, C)**

Sol.

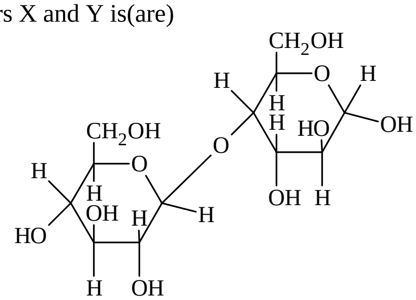


(A) (B) (C) (D)

9. The correct statement(s) about the following sugars X and Y is(are)



X



Y

(A) X is a reducing sugar and Y is a non-reducing sugar
(B) X is a non-reducing sugar and Y is a reducing sugar
(C) The glucosidic linkages in X and Y are α and β, respectively
(D) The glucosidic linkages in X and Y are β and α, respectively

Key. **(B, C)**

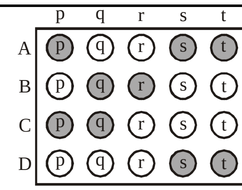
Sol. In "X" the glycosidic linkage is inbetween two anomeric C-atom while in Y it is only with one anomeric carbon, the other one is free. So, "X" will be non-reducing while "Y" will be reducing. Again the glycosidic linkage in X is in between α-glucose and α-fructose, In Y, one of the glucose unit is α.
Hence (B) and (C)

SECTION – III

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



10. Match each of the reactions given in Column I with the corresponding product(s) given in Column II.

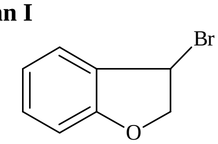
Column I		**Column II**	
(A)	Cu + dil HNO_3	(p)	NO
(B)	Cu + conc HNO_3	(q)	NO_2
(C)	Zn + dil HNO_3	(r)	N_2O
(D)	Zn + conc HNO_3	(s)	$Cu(NO_3)_2$
		(t)	$Zn(NO_3)_2$

Key. **(A – p, s), (B – q, s), (C – r, t), (D – q, t)**

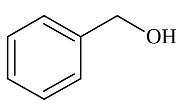
Sol.
$$3Cu + \underset{(dil)}{8HNO_3} \longrightarrow 3Cu(NO_3)_2 + 4H_2O + 2NO\uparrow$$
$$Cu + \underset{(conc)}{4HNO_3} \longrightarrow Cu(NO_3)_2 + 2H_2O + 2NO_2\uparrow$$
$$4Zn + \underset{(dil.)}{10HNO_3} \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O\uparrow$$
$$Zn + \underset{(conc)}{HNO_3} \rightarrow Zn(NO_3)_2 + H_2O + NO_2\uparrow$$

11. Match each of the compounds given in Column I with the reaction(s), that they can undergo, given in Column II.

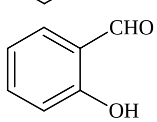
Column I / **Column II**

(A) 

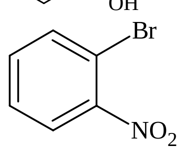
(p) Nucleophilic substitution

(B) 

(q) Elimination

(C) 

(r) Nucleophilic addition

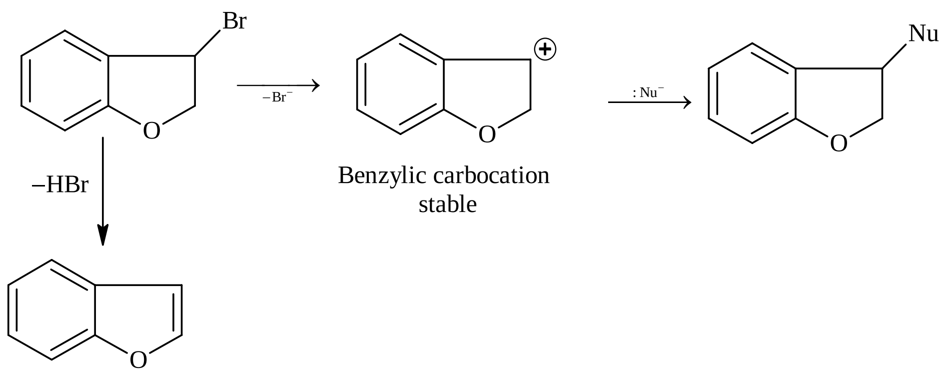
(D) 

(s) Esterification with acetic anhydride

(t) Dehydrogenation

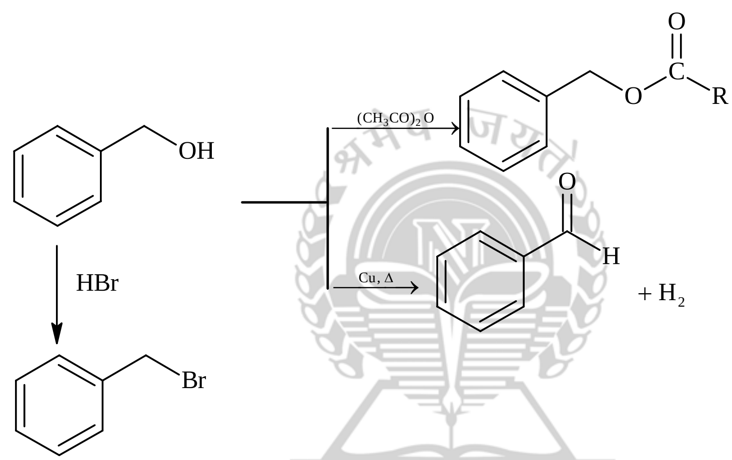
Key. **(A – p, q), (B – p, s, t), (C – r, s), (D – p)**

(A)

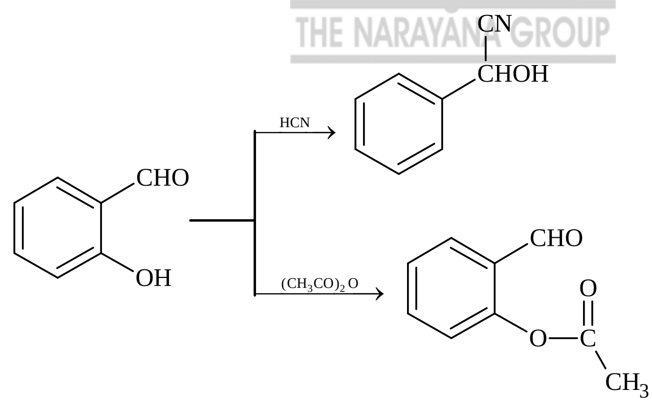


The alkenic d bond is conjugated with the aromatic nucleus

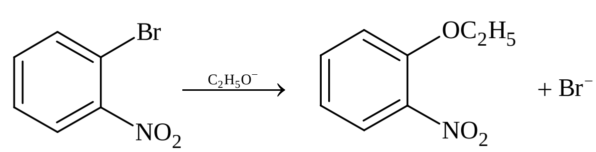
(B)



(C)



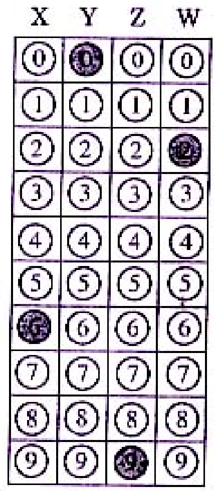
(D)



SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



*12. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in kJ mol^{-1} is

Key. 9 kJ mol^{-1}

Sol. Rise in temperature (293.45 – 298)
= 0.45 K

$\therefore$ Heat evolved = $0.45 \times 2.5 = 1.125$ kJ

$\therefore$ No. of moles $\frac{3.5}{28} = \frac{1}{8}$mol

$\therefore$ Enthalpy of combustion
= 8×1.125
= 9 kJ/moles

*13. At 400K, the root mean square (rms) speed of a gas X (molecular weight = 40) is equal to the most probable speed of gas Y at 60 K. The molecular weight of the gas Y is

Key. 4 $gmol^{-1}$

Sol. $U_{rms} = \sqrt{\frac{3RT}{M}}$

$U_{mp} = \sqrt{\frac{2RT}{M}}$

$\therefore$ From questions

$$\sqrt{\frac{3R \times 400}{40}} = \sqrt{\frac{2R \times 60}{M}}$$

M = 4

*14. The dissociation constant of a substituted benzoic acid at 25°C is 1.0×10^{-4}. The pH of a 0.01 M solution of its sodium salt is

Key. 8

Sol. $pH = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log C$

$= 7 + \frac{1}{2} \times 4 + \frac{1}{2} \log 0.01$

$= 8$

*15. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb$ is

Key. 8

Sol. ${}_{92}U^{238} \longrightarrow {}_{82}Pb^{214}$

No. of α particle = $\frac{238-214}{4}$

$= \frac{24}{4} = 6\alpha$

No. of β particle = 2β

Total particle = 6 + 2 = 8

16. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is

Key. 6

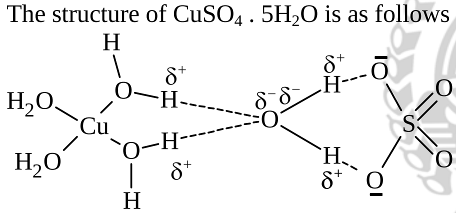
Sol. $MnO_2 + 2KOH + \frac{1}{2}O_2 \longrightarrow K_2MnO_4 + H_2O$

Oxidation state of Mn is +6

17. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 . 5H_2O$ is

Key. 4

Sol. The structure of $CuSO_4 . 5H_2O$ is as follows



That is only four water molecules are coordinated to central Cu^{2+} ion. One H_2O molecule exists H-bonded. Hence answer is 4.

*18. The coordination number of Al in the crystalline state of $AlCl_3$ is

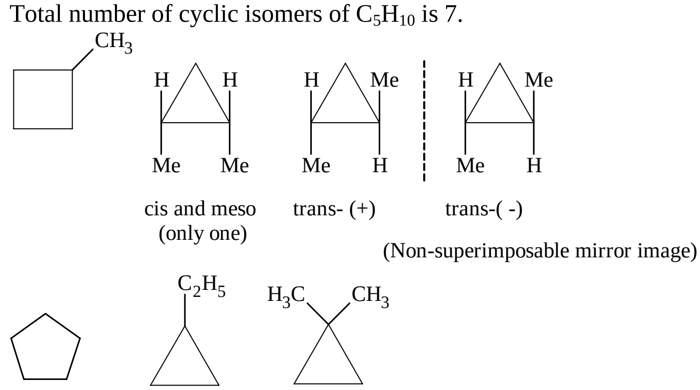
Key. 6

Sol. At low temperature $AlCl_3$ exists a closed packed lattice of Cl^- ions having Al^{3+} ion in octahedral void. Hence C.N. is six.

*19. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

Key. 7

Sol. Total number of cyclic isomers of C_5H_{10} is 7.



PART – II: MATHEMATICS

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

20.

***20.** If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Key (C)

Sol.: $T_n = S_n - S_{n-1} = c[n^2 - (n-1)^2] = c\,(2n-1)$

$\Rightarrow$ Required sum $= c^2 \sum_{r=1}^{n}(4r^2-4r+1) = c^2\left[\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right]$

$= \frac{n(4n^2-1)c^2}{3}$.

21. A line with positive direction cosines passes through the point P(2, -1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$

(C) $\sqrt{3}$ (D) 2

Key (C)

Sol.: Let the line make the angle α with the axes, then we have

$3\cos^2\alpha = 1$ [$\because$ sum of the square's of DC's = 1]

$\cos\alpha = \frac{1}{\sqrt{3}}$ [$\because$ DC's are positive, given]

$\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1\sqrt{3}} = r$ {where r = PQ}

$x = \frac{r}{\sqrt{3}} + 2$, $y = \frac{r}{\sqrt{3}} - 1$, $z = \frac{r}{\sqrt{3}} + 2$

$\frac{2r}{\sqrt{3}} + 4 + \frac{r}{\sqrt{3}} - 1 + \frac{r}{\sqrt{3}} + 2 = 9$

$\frac{4r}{\sqrt{3}} = 4 \Rightarrow r = \sqrt{3}$

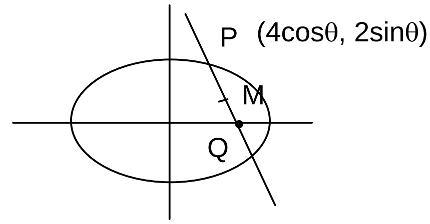
***22.** The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Key (C)

Sol.: $\frac{x^2}{16}+\frac{y^2}{4}=1$



Equation of normal at P

$\frac{4x}{\cos\theta}-\frac{2y}{\sin\theta}=12$

If y = 0, x = 3cosθ ⇒ Q.(3cosθ, 0)

Let mid points PQ be M: (h, k)

⇒ 2h = 7cosθ, 2k = 2sinθ

⇒ locus of 'M'

$\frac{4x^2}{49}+\frac{y^2}{1}=1$ ... (i)

Eccentricity of ellipse is $\sqrt{3}/2$

Equation to Latus rectum x = ± ae ... (ii)

from (i) and (ii) we get y = ± 1/7

Hence the required point (± 2$\sqrt{3}$, ± 1/7)

23. The locus of the orthocentre of the triangle formed by the lines (1 + p)x - py + p (1 + p) = 0, (1 + q) x - qy + q(1 + q) = 0, and y = 0, where p ≠ q, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

Key (D)

Sol.: Let $L_1 \equiv \frac{x}{q}-\frac{y}{1+q}=-1$

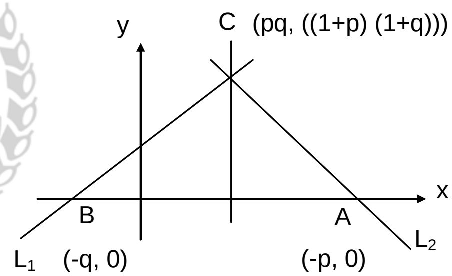
$L_2 = \frac{x}{p}-\frac{y}{1+p}=-1$

Altitudes thro C is x = pq

and altitudes thro B is (1 + p) y + px + pq = 0

Eliminating p and q we get

y = -x so locus is a straight line



SECTION – II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

24.

24. If $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ... , then

(A) $I_n = I_{n+2}$

(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$

(D) $I_n = I_{n+1}$

Key (A, B, C)

24. $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2 ... (i)

$I_n = \int_{0}^{\pi}\frac{\sin nx}{\sin x}\left(\frac{1}{1+\pi^x}+\frac{1}{1+\pi^{-x}}\right)dx$

$\Rightarrow I_n = \int_0^{\pi} \frac{\sin nx}{\sin x} dx$

$I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$

$= \int_0^{\pi} \frac{2\cos\left(\frac{nx+nx+2x}{2}\right).\sin\left(\frac{nx+2x-nx}{2}\right)dx}{\sin x}$

$= \int_0^{\pi} \frac{2\cos(nx+x)\sin x}{\sin x} dx = 2\int_0^{\pi} \cos(n+1)x\,dx$

$= 2\frac{[\sin(n+1)x]_0^{\pi}}{n+1}$

$= 0$

$\therefore I_{n+2} = I_n \ \forall\ n = 0, 1, 2,$

$I_0 = 0$

$I_1 = \pi$

$\sum_{m=1}^{10} I_{2m+1} = 10\pi$

$\sum_{m=1}^{10} I_{2m} = 0$

***25.** An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$ (B) the foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$ (D) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Key (A, B)

25. $x^2 - y^2 = \frac{1}{2}$ (rectangular hyperbola)

eccentricity of rectangular hyperbola = $\sqrt{2}$

$\therefore$ eccentricity of ellipse = $\frac{1}{\sqrt{2}}$

Let equation of an ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{1}{\sqrt{2}} = \sqrt{1 - \frac{b^2}{a^2}}$

$\frac{b^2}{a^2} = \frac{1}{2}$ ... (1)

Let ellipse and hyperbola intersect at (α, β)

$\left(\frac{dy}{dx}\right)_{at\ (\alpha,\beta)}$ for hyperbola = $\left(\frac{\alpha}{\beta}\right)$

$\left(\frac{dy}{dx}\right)_{at\ (\alpha,\beta)}$ for ellipse = $-\frac{b^2}{a^2}\frac{\alpha}{\beta}$

$\therefore \left(\frac{\alpha}{\beta}\right)^2 \frac{b^2}{a^2} = 1$

$\Rightarrow \dfrac{\alpha^2}{\beta^2} = 2$ ... (ii)

$b^2/a^2 = 1/2$

As $\alpha^2 - \beta^2 = 1/2$

$\Rightarrow 2\beta^2 - \beta^2 = 1/2$

$\therefore \beta^2 = 1/2$

from (ii) $\alpha^2 = 1$ ... (iii)

Also $\dfrac{\alpha^2}{a^2} + \dfrac{\beta^2}{b^2} = 1$

$\Rightarrow \dfrac{1}{a^2} + \dfrac{1}{2b^2} = 1$

$\Rightarrow \dfrac{b^2}{a^2} + \dfrac{1}{2} = b^2$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{2} = b^2$

$\therefore b^2 = 1$

Also, $a^2 = 2$

$\therefore$ equation of an ellipse

$\dfrac{x^2}{2} + \dfrac{y^2}{1} = 1$

$\Rightarrow x^2 + 2y^2 = 2$ foci $\equiv (\pm 1, 0)$

26. For the function

$f(x) = x \cos \dfrac{1}{x}, x \geq 1,$

(A) for at least one x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim\limits_{x \to \infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Key (B, C, D)

Sol.: $f(x) = x\cos \dfrac{1}{x}, x \geq 1$

$f'(x) = \cos \dfrac{1}{x} + \dfrac{1}{x} \sin \dfrac{1}{x}$ and $f''(x) = -\dfrac{1}{x^3}\cos\dfrac{1}{x}$

for $x \geq 1 \Rightarrow 0 < \dfrac{1}{x} \leq 1$

$\Rightarrow f''(x) < 0 \ \forall \ x \geq 1$

$\Rightarrow f'(x)$ is strictly decreasing in $[1, \infty)$

$\lim\limits_{x\to\infty} f'(x) = \lim\limits_{x\to\infty} \cos \dfrac{1}{x} + \dfrac{1}{x}\sin\dfrac{1}{x} = \cos 0 + 0 = 1$

Let $g(x) = f(x + 2) - f(x)$

$g'(x) = f'(x + 2) - f'(x) < 0$ (as $f'(x)$ is decreasing)

$\Rightarrow g(x)$ is decreasing

Now, $\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} (x + 2) \cos \dfrac{1}{x+2} - x \cos \dfrac{1}{x}$

$= \lim\limits_{x\to\infty} x\left(\cos\dfrac{1}{x+2} - \cos\dfrac{1}{x}\right) + 2\cos\dfrac{1}{x+2}$

$= \lim\limits_{x\to\infty} 2x\sin\dfrac{x+1}{x^2+2x}.\sin\dfrac{1}{x^2+2x} + 2\cos\dfrac{1}{x+2}$

$$= \lim_{x\to\infty}\left[\frac{2(x^2+x)}{x^2+2x}\left(\frac{\sin\frac{x+1}{x^2+2x}}{\frac{x+1}{x^2+2x}}\right)\sin\frac{1}{x^2+2x}+2\cos\frac{1}{x+2}\right] = 2.1.1.\,0 + 2\,.1 = 2$$

$\Rightarrow g(x) > 2 \ \forall\ x \geq 1$

***27.** The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$

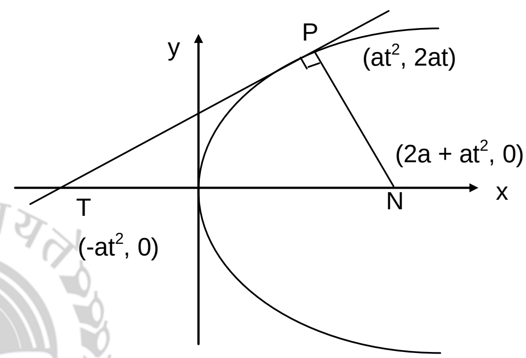
(C) latus rectum is $\frac{2a}{3}$ (D) focus is $(a, 0)$

Key (A, D)

Sol.: Tangent at P, $ty = x + at^2$

Normal at P, $y = -tx + 2at + at^3$

Let centroid be (h, k)



$h = \frac{2a+at^2}{3}$

$k = \frac{2a\,t}{3}$

$\Rightarrow 3ah = 2a^2 + a^2t^2 = 2a^2 + \left(\frac{3k}{2}\right)^2$

$\Rightarrow 12ax = 8a^2 + 9y^2$

$\Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$

Vertex: $\left(\frac{2a}{3}, 0\right)$

Latus rectum = $\frac{4a}{3}$

directrix: $x - \frac{2a}{3} + \frac{a}{3} = 0 \Rightarrow x = a/3$

Focus: $\left(\frac{2a}{3} + \frac{a}{3}, 0\right) = (a, 0)$

***28.** For $0 < \theta < \pi/2$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is/are

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

Key (C, D)

Sol.: $\sum_{m=1}^{6} \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$

$$\Rightarrow \frac{1}{\sin\frac{\pi}{4}}\sum_{m=1}^{6}\frac{\sin\left(\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right)}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$$\Rightarrow \sum_{m=1}^{6}\left(\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right) = 4$$

$\Rightarrow \cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right) = 4$

$\Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \tan^2\theta - 4\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 2 \pm \sqrt{3} \Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

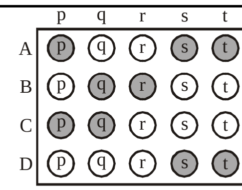
So, (C) and (D) are correct.

SECTION – III

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



29. Match the statements/expressions given in **Column I** with the values given in **Column II.**

	Column - I		**Column - II**
*(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\pi/6$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q)	$\pi/4$
		(r)	$\pi/3$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(s)	$\pi/2$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$	(t)	π

Key **(A-q, s); (B-p, r, s, t); (C-t); (D-r)**

Sol.: (A) $\sin^2 2\theta = 2 - 2\sin^2\theta$

$\Rightarrow 4\sin^2\theta\cos^2\theta = 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta\,(2\sin^2\theta - 1) = 0$

$\Rightarrow \cos\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$

(B) $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$

It is discontinuous when either

$\frac{6x}{\pi} \in I$ or $\frac{3x}{\pi} \in I$ i.e.,

when $x = \pi/6$ or $\pi/3$ or $\pi/2$ or π

(C) Volume = $|[\hat{i}+\hat{j},\ \hat{i}+2\hat{j},\ \hat{i}+\hat{j}+\pi\hat{k}]|$

$= \left| \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} \right| = \pi$

(D) $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c} = 0$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |-\sqrt{3}\vec{c}|^2$

$\Rightarrow 1 + 1 + 2\cos\theta = 3$

$\Rightarrow \cos\theta = 1/2 \;\Rightarrow \theta = \pi/3$

30. Match the statements/expressions given in **Column I** with the values given in **Column II.**

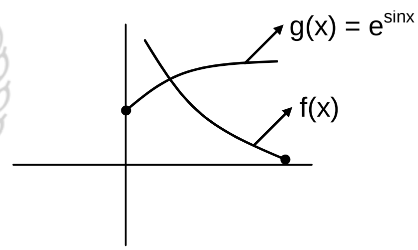
	Column - I		**Column - II**
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
*(B)	Values(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(q)	2
		(r)	3
(C)	Value(s) of k for which |x - 1| + |x - 2| + |x + 1| + |x + 2| = 4k has integer solution(s)	(s)	4
(D)	If y′ = y + 1 and y(0) = 1, then values(s) of y (ln 2)	(t)	5

Key **(A-p), (B-q, s), (C-q, r, s, t), (D-r)**

Sol. (A) $x.e^{\sin x} - \cos x = 0$

$\Rightarrow e^{\sin x} = \frac{\cos x}{x}$

Let $f(x) = \frac{\cos x}{x}$, $g(x) = e^{\sin x}$



According the graph

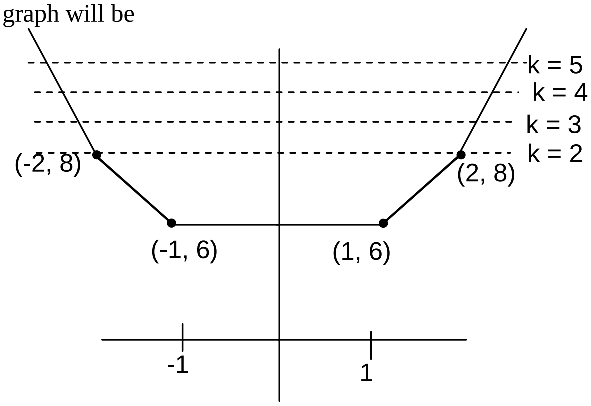
Number of solution is 1.

(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow k = 2, 4$

(C) |x + 1| + |x - 2| + |x + 1| + |x + 2| = 4k

graph will be



So solution is k = 2, 3, 4, 5

(D) $\frac{dy}{dx} = y + 1$

$\int \frac{dy}{y+1} = \int dx$

ln (y + 1) = x + C

(x + c)

$y + 1 = e^{(x+c)}$

$y = c^{x+c} - 1$

x = 0, y = 1

$1 = e^{c} - 1$

$e^{c} = 2$

C = ln 2

$y = e^{x + \ln 2} - 1$

for x = ln2

$e^{2\ln 2} - 1$

y = 3.

SECTION – IV
Integer Answer Type

This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:

31. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \,|x^2 + 20 \le 9x\}$ is

Key 7

Sol.: $A = \{x \,|x^2 + 20 - 9x \le 0\}$

$x^2 - 9x + 20 \le 0$

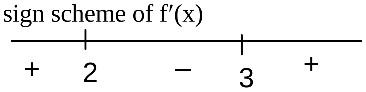
$(x - 5)(x - 4) \le 0$

$x \in [4, 5]$

$f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6(x - 2)(x - 3)$

sign scheme of $f'(x)$



f(x) is strictly increasing in (4, 5)

So, f(5) = 7

32. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

3x - y - z = 0

-3x + z = 0

-3x + 2y + z = 0

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is

Key 7

Sol.:

$3x - y - z = 0$ …..(i)

$-3x + z = 0$ …..(ii)

$-3x + 2y + z = 0$ …..(iii)

Solving (i) & (ii)

y = 0

So 3x – z = 0

z = 3x

Now $x^2 + y^2 + z^2 \le 100$

$x^2 + 9x^2 \le 100$

$|x| \le \sqrt{10}$

$-\sqrt{10} \le x \le \sqrt{10}$

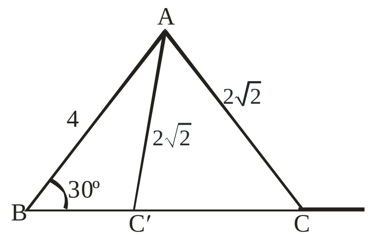
Integral values of ‘x’ are –3, –2, –1, 0, 1, 2, 3

So ‘7’ points are there.

***33.** Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

Key 4

Sol.:



Using sine rule in ΔABC

$$\frac{\sin C}{4} = \frac{\sin 30°}{2\sqrt{2}} \Rightarrow C = 45°$$

∴ ∠AC′C = 45º and ∠C′AC = 90º

Difference of area of ΔABC & ΔABC′ is area of ΔACC′

$$= \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) = 4$$

34. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)$ = 2. Then the value of p(2) is

Key 0

Sol.: Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

Given $\lim_{x\to 0}\left(1+\frac{P(x)}{x^2}\right) = 2$

Limit exist only if, d = e = 0

$$\lim_{x\to 0}[1 + ax^2 + bx + c] = 2$$

⇒ c + 1 = 2

⇒ c = 1

$P(x) = ax^4 + bx^3 + x^2$

$P'(x) = 4ax^3 + 3bx^2 + 2x$

$= x(4ax^2 + 3bx + 2)$

Note: $4ax^2 + 3bx + 2 \equiv \lambda(x - 1)(x - 2) = \lambda(x^2 - 3x + 2)$

$\Rightarrow \lambda = 1,\ a = \frac{1}{4},\ b = -1$

$\Rightarrow \quad P(x) = \frac{1}{4}x^4 - x^3 + x^2$

$\therefore \quad P(2) = \frac{1}{4}2^4 - 2^3 + 2^2$

$= 4 - 8 + 4 = 0$

35. Let f : R → R be a continuous function which satisfies

$f(x) = \int_0^x f(t)\,dt$. Then the value of f(ln 5) is

Key 0

Sol.: $f'(x) = f(x)$

⇒ $f(x) = ce^x$

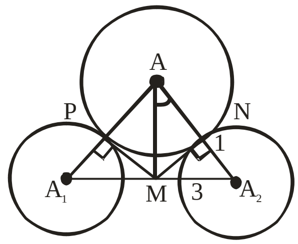
⇒ c = 0 because f(0) = 0

∴ f(ln 5) = 0

***36.** The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 extrenally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

Key 8

Sol.:



$PM = PN = \sqrt{9-1} = 2\sqrt{2}$

Clearly AM perpendicular to A_1A_2

Let radius of circle C = r

In ΔAMA_2

$(AM)^2 + 9 = (r + 1)^2$ …..(i)

In ΔAMN

$(AM)^2 = r^2 + 8$

from (i) and (ii) r = 8.

***37.** The smallest value of k, fow which both the roots of the equation

$x^2 - 8kx + 16(k^2 - k + 1) = 0$

are real, distinct and have values at least 4, is

Key 2

Sol.: For the root to be real & distinct and having minimum value, following points should hold :

(1) $D > 0$

(2) $-\frac{b}{2a} > 4$

(3) $f(4) \geq 0$

(1) $64k^2 - 64(k^2 - k + 1) > 0$

$\Rightarrow k > 1$

(2) $\frac{8k}{2} > 4 \Rightarrow k > 1$

(3) $16 - 32k + 16k^2 - 16k + 16 \geq 0$

$(k - 2)(k - 1) \geq 0$

$k \geq 2$ or $k \leq 1$

Taking the intersection the required solution is $k \geq 2$.

Alternate

$(x - 4k)^2 = 16(k - 1)$

$\Rightarrow k \geq 1$

$\Rightarrow x = 4k \pm 4\sqrt{k-1}$

real solutions for $k \geq 1$

Note : $4k + 4\sqrt{k-1} \geq 4$ always true for $k \geq 1$

Now, $4k - 4\sqrt{k-1} \geq 4$

$\Rightarrow k - 1 \geq \sqrt{k-1}$

$\Rightarrow (k - 1)^2 \geq (k - 1)$

$\Rightarrow (k - 1)(k - 2) \geq 0$

$\Rightarrow k \geq 2$ for distinct & both roots more than 4.

38. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Key 2

Sol.: $f(x) = x^3 + e^{x/2}$

$g(x) = f^{-1}(x)$

$f(g(x)) = x \quad f(0) = 1 \Rightarrow f^{-1}(1) = 0$

$\Rightarrow g(1) = 0$

$f'(g(x)).g'(x) = 1$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(1) = \frac{1}{f'(g(1))}$$

$$\Rightarrow \quad = \frac{1}{f'(0)}$$

$$f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$$

$$f'(0) = \frac{1}{2}$$

$$g'(1) = 2$$

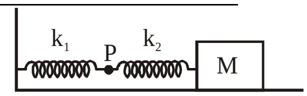
PART – III: PHYSICS

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

*39. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$.

Key, (D)

Sol. $x_1 + x_2 = A$

$k_1 x_1 = k_2 x_2$

$\therefore$ amplitude of point $P = \frac{k_2 A}{k_1 + k_2}$.

$\therefore$ (D) is correct.

*40. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y–axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x–axis with constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, form the y–axis is

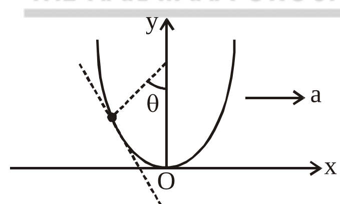
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$

(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$.

Key (B)

Sol. $\tan\theta = \frac{a}{g}$

slope of tangent

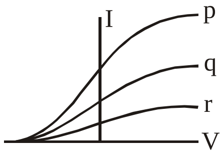


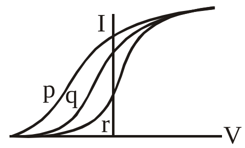
$$= \tan(\pi - \theta) = -\tan\theta = \frac{dy}{dx} = 2kx$$

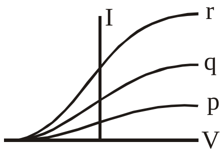
$$\Rightarrow \quad x = -\frac{a}{2kg}$$

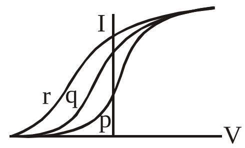
$\therefore$ (B) is correct.

41. Photoelectric effect experiments are performed using three different metal plates p, q and r having work function $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelength of 550 nm and 350 nm with equal intensities illuminates each of the plates. The correct I–V graph for the experiment is

(A) 

(B) 

(C) 

(D) 

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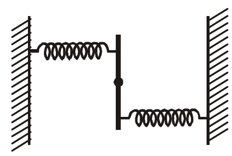
Key. (A)

Sol. The work function for P is smallest.

$\therefore$ stopping potential for P is largest. Secondly, not all wavelengths will be able to eject photoelectron from all three i.e., the saturation current will be different.

$\therefore$ (A) is correct.

*42. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

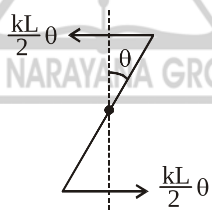


(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$.

Key. (C)

Sol. Restoring torque $\tau_r = -k\frac{L}{2}\theta\cdot\frac{L}{2}\times 2$



$\Rightarrow \frac{ML^2}{12}\alpha = -\frac{k}{2}L^2\theta \Rightarrow \alpha = -\frac{6k}{M}\theta$

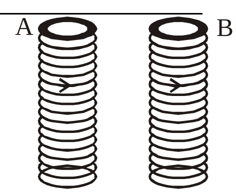
$\therefore f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ $\therefore$ (C) is correct.

SECTION – II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

43. Two metallic rings A and B, identical in shape and size but having different resistivity ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$ (B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$ (D) $\rho_A < \rho_B$ and $m_A < m_B$.

Key. (B, D)

Sol. $q = \frac{\Delta\phi}{R} \propto \frac{1}{\rho}$ …(i)

$\int I\ell B_\gamma dt = mv$

$\Rightarrow \ell B_\gamma q = mv$

$\Rightarrow v \propto \frac{q}{m}$ …(ii)

Also $v^2 \propto h$ …(iii)

From (i), (ii) and (iii)

$(m\rho)_A < (m\rho)_B$

$\therefore$ (B) and (D) are correct.

*44. A student performed the experiment to measure the speed of sound in air using resonance air–column method. Two resonances in the air–column were obtained by lowering the water level. The resonance with the shorter air–column is the first resonance and that with the longer air–column is the second resonance. Then,

(A) the intensity of the sound heard at the first resonance was more than that at the second resonance
(B) the prongs of the tunning fork were kept in a horizontal plane above the resonance tube
(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(D) the length of the air–column at the first resonance was somewhat shorter than $1/4^{th}$ of the wavelength of the sound in air.

Key. (A, D)

Sol. As length of air–column increases intensity decreases.

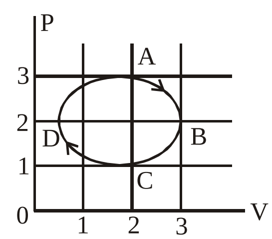
Hence (A) is correct.

$\ell + e = \frac{\lambda}{4} \Rightarrow \ell < \frac{\lambda}{4}$.

Hence (D) is correct.

*45. The figure shows the P–V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi–circle and CDA is half of an ellipse. Then,

(A) the process during the path $A \to B$ is isothermal
(B) heat flows out of the gas during then path $B \to C \to D$
(C) work done during the path $A \to B \to C$ is zero
(D) positive work is done by the gas in the cycle ABCDA.



Key. (B, D)

Sol. Temperature at B > temperature at D

$\therefore$ ΔU is negative (for $B \to C \to D$)

Also W is negative (for $B \to C \to D$)

Tracing is clockwise on PV diagram.

$\therefore$ W is positive.

$\therefore$ (B) and (D) are correct.

46. Under the influence of the Coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbit. Find out the correct statement (s)

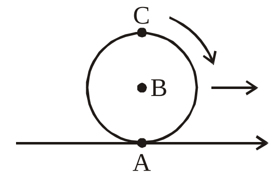
(A) the angular momentum of the charge –q is constant
(B) the linear momentum of the charge –q is constant
(C) the angular velocity of the charge –q is constant
(D) the linear speed of the charge –q is constant.

Key. (A)

Sol. Force is central.

So, option (A) is correct.

*47. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

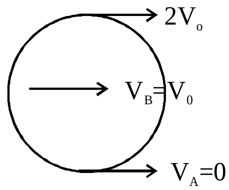


(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$.

Key. (B, C)

Sol.



$\vec{V}_B - \vec{V}_A = \vec{V}_o$

$\vec{V}_C - \vec{V}_B = \vec{V}_o$

$|\vec{V}_C - \vec{V}_A| = 2V_o$

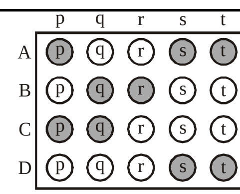
$|\vec{V}_C - \vec{V}_B| = V_o$

So, options (B) and (C) are correct.

SECTION – III
Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:

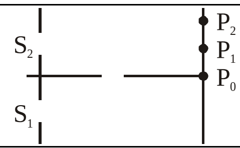
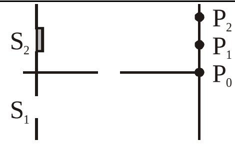
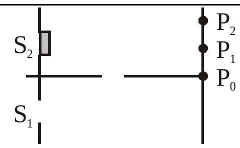
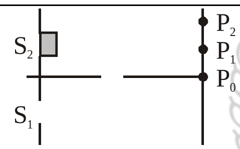


48. Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II.

	Column I	**Column II**	
(A)	The energy of the system is increased	(p)	System : A capacitor, initially uncharged. Process : It is connected to a battery.
(B)	Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q)	System : A gas in an adiabatic container fitted with an adiabatic piston. Process : The gas is compressed by pushing the piston.
(C)	Internal energy of the system is converted into its mechanical	(r)	System : A gas in a rigid container. Process : A gas gets cooled due to colder atmosphere surrounding it.
(D)	Mass of the system is decreased	(s)	System : A heavy nucleus, initially at rest. Process : The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted.
		(t)	System : A resistive wire loop. Process : The loop is placed in a time varying magnetic field perpendicular to its plane.

Key. (A) – (p, q, t) [When current will pass through loop its temperature will increase]
(B) – (q)
(C) – (s)
(D) – (s)

49. Column I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 = S_2P_1 = \lambda/2$ and $S_1P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by I(P). Match each situation given in Column I with the statement(s) in Column II valid for that situation.

	Column I		**Column II**
(A)		(p)	$\delta(P_0) = 0$
(B)	 $(\mu-1)t\lambda/4$	(q)	$\delta(P_1) = 0$
(C)	 $(\mu-1)t = \lambda/2$	(r)	$I(P_1) = 0$
(D)	 $(\mu-1)t = 3\lambda/4$	(s)	$I(P_0) > I(P_1)$
		(t)	$I(P_2) > I(P_1)$

Key. (A) – (p, s), (B) – (q), (C) – (t), (D) – (r, s, t)

Sol. (A) – (p, s)

$I(P_1) = I_{max}\cos^2\frac{\pi}{4}$

$I(P_2) = I_{max}\cos^2\frac{\pi}{3}$

(B) – (q)
P_1 is central maxima

(C) – (t)
P_o is minima in this case
and At, P_2

path difference is $\frac{\lambda}{6}$ $\left(\phi = \frac{\pi}{3}\right)$

while at P_1 path difference is $\frac{\lambda}{4}$ $\left(\phi = \frac{\pi}{2}\right)$.

(D) – (r, s)

At P_o $\Delta x = \frac{3x}{4}$ $\Rightarrow \phi = \frac{3\pi}{2}$

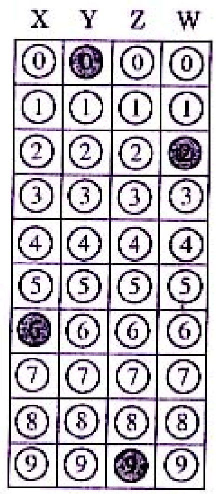
At P_1 $\Delta x = \frac{x}{2}$ $\Rightarrow I(P_1) = 0$

At P_2 $\Delta x = \frac{15x}{12}$ $\Rightarrow \phi = \frac{5\pi}{6}$

SECTION – IV

Integer Answer Type

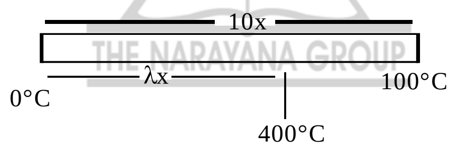
This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



*50. A metal rod AB of length 10x has its one end A in ice at 0ºC and the other end B in water at 100ºC. If a point P on the rod is maintained at 400ºC, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ.
[Neglect any heat loss to the surrounding.]

Key. 9.

Sol. $\left(\frac{\Delta Q}{\Delta t}\right)_\ell = \frac{300}{\frac{10x-\lambda x}{KA}} = mL_V$ …(1)



and $\left(\frac{\Delta Q}{\Delta t}\right)_w = \frac{400}{\frac{\lambda x}{KA}} = mL_f$ …(2)

dividing $\frac{300}{10x-\lambda x}\times\frac{\lambda x}{400} = \frac{L_v}{L_f}$

$\Rightarrow \frac{3}{4}\frac{\lambda}{(10-\lambda)} = \frac{540}{80}$

$\lambda = 9(10-\lambda)$

$10\lambda = 90$

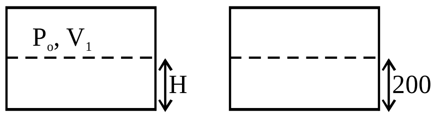
$\Rightarrow \lambda = 9$

*51. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel become steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Key. 6 mm.

Sol.



$P_1 V_1 = P_2 V_2$

$$P_o \times A(0.5 - H) = (P_o - \rho g \times 0.2)A \times 0.3$$

$$\Rightarrow \quad 10^5(0.5 - H) = (10^5 - 2 \times 10^3) \times 0.3$$

$$\Rightarrow \quad 100(0.5 - H) = (100 - 2) \times 0.3$$

$$0.5 - H = \frac{29.4}{100}$$

$\Rightarrow$ H = 0.5 - .294

$\Rightarrow$ H = 0.206

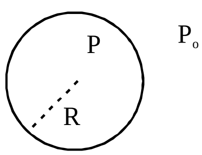
$\Rightarrow$ H = 206 mm

So fall in height = 6 mm.

*52. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap–water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Key. 6.

Sol.



then $P = P_o + \frac{4S}{r}$

Now $P \times \frac{4}{3}\pi r^3 = nR_g T$

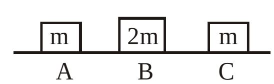
$$\Rightarrow \quad \left(P_o + \frac{4S}{r}\right)\frac{4}{3}\pi r^3 = nR_g T$$

For two bubbles

$$\frac{\left(P_o + \frac{4S}{r_A}\right) r_A^3}{\left(P_o + \frac{4S}{r_B}\right) r_B^3} = \frac{n_A}{n_B}$$

$$\frac{\left(8 + \frac{4 \times 0.04}{2 \times 10^{-2}}\right)\left(2 \times 10^{-2}\right)^3}{\left(8 + \frac{4 \times 0.04}{4 \times 10^{-2}}\right)\left(4 \times 10^{-2}\right)^3} = \frac{n_A}{n_B} \quad \Rightarrow \quad \frac{n_B}{n_A} = 6$$

*53. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.



Key. 4 m/s.

Sol. For collision between A and B

$m \times 9 + 2m \times 0 = mV_1 + 2mV_2$

$\Rightarrow \quad 9 = V_1 + 2V_2 \qquad \ldots(1)$

$e = 1$

So

$9 = V_2 - V_1 \qquad \ldots(2)$

Solving (1) and (2)

$V_2 = 6$ m/s

For collision between B and D

$2m \times 6 = (2m + m)^{v}$

$\Rightarrow \quad v = \dfrac{2 \times 6}{3} = 4$ m/s

54. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\dfrac{\mu_0 I}{48\pi x}\right)$, find the value of k.

Key. K = 7.

Sol. d = 4x cos 37

$= 4x \times \dfrac{3}{5}$

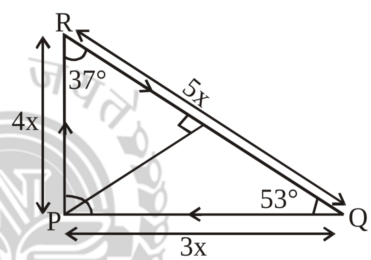
Now,

$B_P = \dfrac{\mu_o I}{4\pi d}\left[\sin 37 + \sin 53\right]$

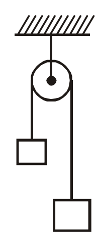
$= \dfrac{\mu_o I}{4\pi \dfrac{12x}{5}}\left[\dfrac{7}{5}\right] = \dfrac{7}{48}\dfrac{\mu_o I}{\pi x}$

$= 7\left(\dfrac{\mu_o I}{48\pi x}\right)$

So, K = 7



*55. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two block of masses 0.36 and 0.72 kg. Taking g = 10 m/s^2, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Key. 8 J.

Sol. $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{0.36 \times 10}{1.08} = \dfrac{10}{3} m/s^2$

Displacement

$S = \dfrac{1}{2}gt^2$

$= \dfrac{1}{2} \times \dfrac{10}{3} \times (1)^2 = \dfrac{5}{3} m$

$T = \dfrac{2m_1 m_2 g}{m_1 + m_2} = \dfrac{2 \times 0.36 \times 0.72 \times 10}{3 \times 0.36} N$

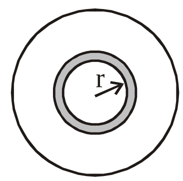
$= 2 \times 2.4 N = 4.8 N$

$W = \vec{T}.\vec{S} = 1.6 \times \frac{5}{3} = 4.8 \times \frac{5}{3} = 8J$

56. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = \kappa r^a$, where κ and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at r = R, find the value of a.

Key. 2.

Sol. for the element



$dq = \rho dV$

$= kr^{\alpha} 4\pi r^2 dr$

$q = \int dq = \int kr^{\alpha} 4\pi r^2 dr$

$q = \frac{k4\pi r^{\alpha+3}}{\alpha+3}$

$E_r = \left[\frac{K4\pi r^{\alpha+3}}{\alpha+3}\right]\frac{1}{r^2}$

$E_R = \left[\frac{K4\pi r^{\alpha+3}}{\alpha+3}\right]\frac{1}{R^2}$, $\frac{E_r}{E_R} = \frac{1}{8} = \frac{r^{\alpha+1}}{R^{\alpha+1}}$ (where r = R/2)

$\Rightarrow \quad \alpha = 2$

*57. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

Key. 5 cm.

Sol. $\ell = 20m$, m = 1 gm, T = 0.5 N

f = 100 Hz

$\mu = \frac{1\times10^{-3}}{20\times10^{-2}} = \frac{1}{2}\times10^{-2}$

$V = \sqrt{\frac{0.5}{0.5\times10^{-2}}} = 10m/s$, $\lambda = 0.1$

$\frac{\lambda}{2} = 0.05m = 5cm$.

MARKING SCHEME

PAPER – I

1. For each questions in **Section I**, you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (–1) mark** will be awarded
2. For each question in **Section II**, you will be **awarded 4 marks** if you have darkened all the bubble(s) corresponding to the correct choice(s) for the answer, and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
3. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
4. For each question in **Section IV**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is **no negative marking** for incorrect answer(s) for this section.

PAPER – II

1. For each question in **Section – I**, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (–1) mark** will be awarded.
2. For each question in **Section II**, you will be **awarded 4 marks s** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
3. For each question in **Section III**, you will be **awarded 2** for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is no negative marking for incorrect answer(s) for this section.
4. For each question in **Section IV**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer, and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.

Now that the JEE is over, continue the same sprit into AIEEE, BITS and so on ....