

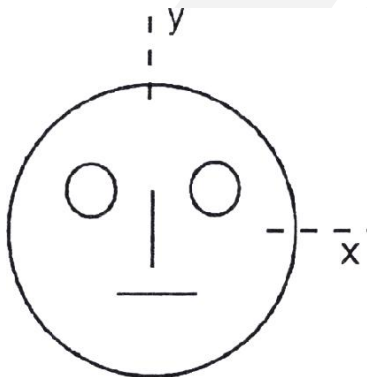
TEST PAPER 1 OF 12 -April -09

PART I - Physics

SECTION I - Straight Objective Type

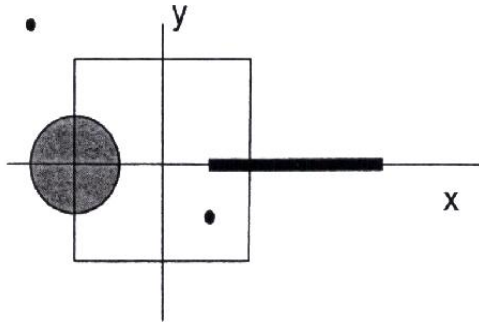
41. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is  $m$ . The mass of the ink used to draw the outer circle is  $6m$ .

The coordinates of the centres of the different parts are : outer circle  $(0, 0)$  left inner circle  $(-a, a)$ , right inner circle  $(a, a)$ , vertical line  $(0,0)$  and horizontal line  $(0, -a)$ . The  $y$ -coordinate of the centre of mass of the ink in this drawing is



- (A)  $a/10$                       (B)  $a/8$                       (C)  $a/12$                       (D)  $a/3$
41. (A)  $[(6m \cdot 0 + m \cdot a + m \cdot a + m \cdot 0 + m \cdot (-a))] / 10m = a/10$ .
42. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time.  $I_1$  and  $I_2$  are the currents in the segments  $ab$  and  $cd$ . Then,





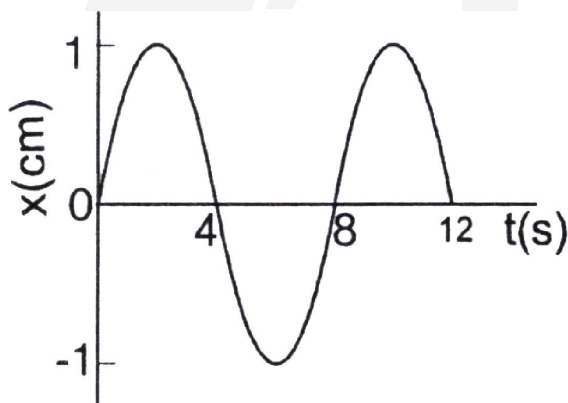
- (A)  $-2C / \epsilon_0$       (B)  $2C / \epsilon_0$       (C)  $10C / \epsilon_0$       (D)  $12C / \epsilon_0$

44. (A) Charge enclosed =  $1/2$  disc +  $1/4$  rod +  $q_{-7}$   
 $= -2C$   
 $\therefore \phi = q_E / \epsilon_0 = -2C / \epsilon_0$

45. Three concentric metallic spherical shells of radii  $R, 2R, 3R$ , are given charge  $Q_1, Q_2, Q_3$ , respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells  $Q_1 : Q_2 : Q_3$ , is  
 (A)  $1 : 2 : 3$       (B)  $1 : 3 : 5$       (C)  $1 : 4 : 9$       (D)  $1 : 8 : 18$

45. (B)  $Q_1 / R^2 = (Q_1 + Q_2) / 4R^2 = (Q_1 + Q_2 + Q_3) / 9R^2$   
 $\Rightarrow Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

46. The  $x-t$  graph of particle undergoing simple harmonic motion is shown below. The acceleration of the particle at  $t = 4/3$  s is



- (A)  $(\sqrt{3} / 32)\pi^2 \text{ cm} / \text{s}^2$       (B)  $(-\pi^2 / 32) \text{ cm} / \text{s}^2$   
 (C)  $(\pi^2 / 32) \text{ cm} / \text{s}^2$       (D)  $(-\pi^3 / 32) \pi^2 \text{ cm} / \text{s}^2$

46. (D)  $x = 1 \sin(\pi t / 4)$ .  $\Rightarrow a = -(\pi^2 / 16) \sin(\pi t / 4)$ ,  
 $\therefore$  At  $t = (4/3)$  s,  $a = -\sqrt{3}\pi^2 / 32 \text{ cm} \text{ s}^{-2}$

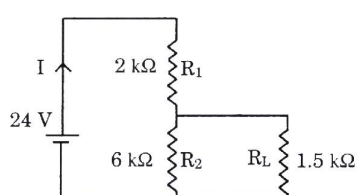
47. A ball is dropped from height of 20 m above the surface of water in a lake. The refractive index of water is  $4/3$ . A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as  
 (A) 9 m/s      (B) 12 m/s      (C) 16 m/s      (D) 21.33 m/s

47. (C)  $v_{\text{actual}} = \sqrt{2g\Delta h} = 12 \text{ ms}^{-1}$        $\therefore v_{\text{app}} = \mu v = 16 \text{ ms}^{-1}$

48. A block of base  $10 \text{ cm} \times 10 \text{ cm}$  and height  $15 \text{ cm}$  is kept on an inclined plane. The coefficient of friction between them is  $\sqrt{3}$ . The inclination  $\theta$  of this inclined plane from the horizontal plane is gradually increased from  $0^\circ$ . Then
- (A) at  $\theta = 30^\circ$ , the block will start sliding down the plane  
 (B) the block will remain at rest on the plane up to certain  $\theta$  and then it will topple  
 (C) at  $\theta = 60^\circ$ , the block will start sliding down the plane and continue to do so at higher angles  
 (D) at  $\theta = 60^\circ$ , the block will start sliding down the plane and on further increasing  $\theta$ , it will topple at certain  $\theta$
48. **(B)**  $\theta_{\text{slide}} = \tan^{-1} \mu = 60^\circ$ ,  $\theta_{\text{topple}} = \tan^{-1} (b / h) = \tan^{-1} (2 / 3)$   
 So toppling before sliding.

### SECTION II - Multiple Type

49. For the circuit shown in the figure



- (A) the current  $I$  through the battery is  $7.5 \text{ mA}$   
 (B) the potential difference across  $R_L$  is  $18 \text{ V}$   
 (C) ratio of powers dissipated in  $R_1$  and  $R_2$  is  $3$   
 (D) if  $R_1$  and  $R_2$  are interchanged, magnitude of the power dissipated in  $R_L$  will decrease by a factor of  $9$
49. **(AD)**  $R_{2L} = 1.2 \text{ k}\Omega$ ,  $R_{\text{eq}} = 3.2 \text{ k}\Omega$ ,  $i_b = 7.5 \text{ mA}$ ,  
 $V_1 = 15 \text{ V}$ ,  $V_2 = V_L = 9 \text{ V}$ ,  $P_1 = 112.5 \text{ mW}$ ,  $P_2 = 13.5 \text{ mW}$ ,  $P_L = 54 \text{ mW}$   
 If interchanged,  $R_{\text{eq}} = 48 / 7 \text{ k}\Omega$ ,  $i_b = 3.5 \text{ mA}$   
 $i_L = 2 \text{ mA} \Rightarrow P_L = 6 \text{ mW}$ .
50.  $C_v$  and  $C_p$  denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then
- (A)  $C_p - C_v$  is larger for a diatomic ideal gas than for a monoatomic ideal gas  
 (B)  $C_p + C_v$  is larger for a diatomic ideal gas than for a monoatomic ideal gas  
 (C)  $C_p / C_v$  is larger for a diatomic ideal gas than for a monoatomic ideal gas  
 (D)  $C_p \cdot C_v$  is larger for a diatomic ideal gas than for a monoatomic ideal gas
50. **(BD)**  $C_p - C_v = R$  for all ideal gases.  
 Both  $C_p$  &  $C_v$  increase with atomicity  
 $\Rightarrow C_p / C_v$  reduces with atomicity.
51. A student performed the experiment of determination of focal length of a concave mirror by  $u-v$  method using an optical bench of length  $1.5 \text{ meter}$ . The focal length of the mirror used is  $24 \text{ cm}$ . The maximum error in the location of the image can be  $0.2 \text{ cm}$ . The 5 sets of  $(u, v)$  values recorded by the student (in  $\text{cm}$ ) are  $(42, 56)$   $(48, 48)$ ,  $(60, 40)$ ,  $(66, 33)$   $(78, 39)$ . The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)
- (A)  $(42, 56)$                       (B)  $(48, 48)$                       (C)  $(66, 38)$                       (D)  $(78, 39)$

51. (CD)  $u = 42, v = 168 / 3 = 56$   
 $u = 66, v = 264 / 7 = 37.7 \Rightarrow 33$  not possible  
 $u = 78, v = 104 / 3 = 34.7 \Rightarrow 39$  not possible
52. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that  
 (A) linear momentum of the system does not change in time  
 (B) kinetic energy of the system does not change in time  
 (C) angular momentum of the system does not change in time  
 (D) potential energy of the system does not change in time
52. (A)  $KE$  &  $PE$  can change (e.g blocks at ends of spring)  
 $L$  can change (if resultant is net torque)

**SECTION IV - Linked Comprehension Type**  
**Paragraph for Questions 53 to 55**

When a particle is restricted to move along  $x$ -axis between  $x = 0$  and  $x = a$ , where  $a$  is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends  $x = 0$  and  $x = a$ . The wavelength of this standing wave is related to the linear momentum  $p$  of the particle according to the de Broglie relation. The energy of the particle of mass  $m$  is related to its linear momentum as

$E = \frac{p^2}{2m}$ . Thus, the energy of the particle can be denoted by a quantum number ' $n$ ' taking values 1, 2, 3 ... ( $n = 1$ , called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line  $x = 0$  to  $x = a$ . Take  $h = 6.6 \times 10^{-34}$  Js and  $e = 1.6 \times 10^{-19}$  C.

53. The allowed energy for the particle for a particular value of  $n$  is proportional to  
 (A)  $a^{-2}$  (B)  $a^{-3/2}$  (C)  $a^{-1}$  (D)  $a^2$
53. (A) For  $n$  loop,  $\lambda = 2a / n \Rightarrow p = h / \lambda = nh / 2a$   
 $\Rightarrow E = p^2 / 2m = n^2 h^2 / 8a^2 m$ .
54. If the mass of the particle is  $m = 1.0 \times 10^{-30}$  kg and  $a = 6.6$  nm, the energy of the particle in its ground state is closest to  
 (A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV
54. (B)  $E = (h / a)^2 / 8m$   
 $= 10^{-50} / 8 \cdot 10^{-30} = 1.25 \times 10^{-21} \text{ J} \approx 8 \text{ meV}$ .
55. The speed of the particle, that can take discrete values, is proportional to  
 (A)  $n^{-3/2}$  (B)  $n^{-1}$  (C)  $n^{1/2}$  (D)  $n$
55. (D) Since  $p = nh / 2a, v = nh / 2am$

**Paragraph for Questions 56 to 58**

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen,  ${}^2_1H$ , known as deuteron and denoted by  $D$ , can be thought of as a candidate for fusion reactor. The  $D - D$  reaction is  ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + \text{energy}$ . In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of  ${}^2_1H$  nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time  $t_0$  before the particles fly away from the core. If  $n$  is the density (number/volume) of deuterons, the product  $nt_0$  is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than  $5 \times 10^{14} \text{ s cm}^{-3}$ . It may be helpful to use the following: Boltzmann constant  $k = 8.6 \times 10^{-5} \text{ eV/K}$ ;

$$\frac{e^2}{4\pi\epsilon_0} = 1.44 \times 10^{-9} \text{ eVm.}$$

56. In the core of nuclear fusion reactor, the gas becomes plasma because of  
(A) strong nuclear force acting between the deuterons  
(B) Coulomb force acting between the deuterons  
(C) Coulomb force acting between deuteron-electron pairs  
(D) the high temperature maintained inside the reactor core
56. **(D)**
57. Assume that two deuteron nuclei in the core of fusion reactor at temperature  $T$  are moving towards each other, each with kinetic energy  $1.5 kT$ , when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature  $T$  required for them to reach a separation of  $4 \times 10^{-15} \text{ m}$  is in the range  
(A)  $1.0 \times 10^9 \text{ K} < T < 2.0 \times 10^9 \text{ K}$       (B)  $2.0 \times 10^9 \text{ K} < T < 3.0 \times 10^9 \text{ K}$   
(C)  $3.0 \times 10^9 \text{ K} < T < 4.0 \times 10^9 \text{ K}$       (D)  $4.0 \times 10^9 \text{ K} < T < 5.0 \times 10^9 \text{ K}$
57. **(A)**  $PE = \frac{e^2}{4\pi\epsilon_0 d} = 3.6 \times 10^5 \text{ eV}$ .
- $\therefore 3kT \approx 3.6 \times 10^5 \text{ eV} \Rightarrow T \approx \frac{12}{8.6} \times 10^9 \text{ K} \approx 1.4 \times 10^9 \text{ K}$
58. Results of calculations for four different designs of a fusion reactor using  $D - D$  reaction are given below. Which of these is most promising based on Lawson criterion?  
(A) deuteron density =  $2.0 \times 10^{12} \text{ cm}^{-3}$ , confinement time =  $5.0 \times 10^{-3} \text{ s}$   
(B) deuteron density =  $8.0 \times 10^{14} \text{ cm}^{-3}$ , confinement time =  $9.0 \times 10^{-1} \text{ s}$   
(C) deuteron density =  $4.0 \times 10^{23} \text{ cm}^{-3}$ , confinement time =  $1.0 \times 10^{-11} \text{ s}$   
(D) deuteron density =  $1.0 \times 10^{24} \text{ cm}^{-3}$ , confinement time =  $4.0 \times 10^{-12} \text{ s}$
58. **(B)**  $L_1 = 10^{10}$ ,  $L_2 = 7.2 \times 10^{14}$ ,  $L_3 = 4 \times 10^{12}$ ,  $L_4 = 4 \times 10^{12}$

### SECTION IV - Matrix Match Type

This section contains 2 questions. Each questions contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled  $p, q, r, s$  and  $t$ . Any given statement in **Column I** can have correct matching with **one or more** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are  $A - p, s$  and  $t$ ;  $B - q$  and  $r$ ;  $C - p$  and  $q$ ; and  $D - s$  and  $t$ ; then the correct darkening of bubbles will look like the following.

59. **Column II** shows five systems in which two objects are labelled as  $X$  and  $Y$ . Also in each case a point  $P$  is shown. **Column I** gives some statements about  $X$  and / or  $Y$  Match these statements to the appropriate system(s) from **Column II**.

#### Column I

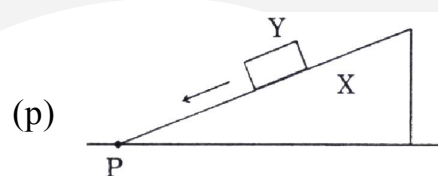
(A) the force exerted by  $X$  on  $Y$

has a magnitude  $Mg$ .

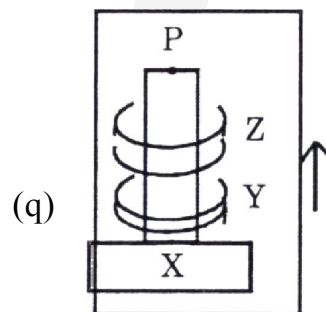
(B) The gravitational potential energy

of  $X$  is continuously increasing

#### Column II

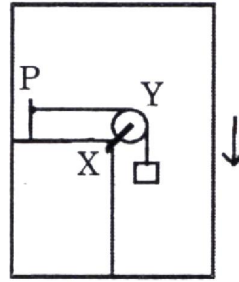


Block  $Y$  of mass  $M$  left on a fixed inclined plane  $X$ , slides on it with a constant velocity



Two ring magnets  $Y$  and  $Z$ , each of mass  $M$ , are kept in frictionless vertical plastic stand so that they repel each other.  $Y$  rests on the base  $X$  and  $Z$  hangs in air in equilibrium.  $P$  is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(C) Mechanical energy of the system (r)

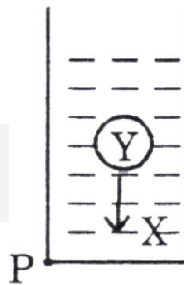


$X + Y$  is continuously decreasing.

A pulley  $Y$  of mass  $m_0$  is fixed to a table  $X$  through a clamp  $X$ . A block of mass  $M$  hangs from a string that goes over the pulley and is fixed at point  $P$  of the table. The whole system is kept in a lift that is going down with a constant velocity.

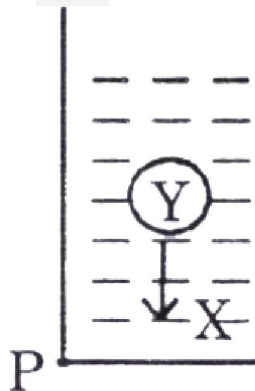
(D) The torque of the weight of  $Y$  (s)

about point  $P$  is zero



A sphere  $Y$  of mass  $M$  is put in a nonviscous liquid  $X$  kept in a container at rest. The sphere is released and it moves down in the liquid.

(t)



A sphere  $Y$  of mass  $M$  is falling with its terminal velocity in a viscous liquid  $X$  kept in a container,

59. (p) Constant velocity  $\Rightarrow$  Net force = 0  $\Rightarrow F_X = Mg$ .

Since  $X$  fixed,  $PE$  does not change.

Since  $Y$  loses  $PE$  ( $KE$  constant),  $ME$  reduces.

Since weight does not pass through  $P$ ,  $\tau \neq 0$

(q) Force on  $Y$  by  $Z = Mg \downarrow \therefore F_X = 2Mg$ .

Since going up,  $PE_X \uparrow$

Since going up,  $ME \uparrow$   
 Since weight thro' centre,  $\tau = 0$ .

(r)  $F_x =$  Resultant of tensions ( $mg$  each) &  $m_0g \neq Mg$  going down,  $PE_x \downarrow$   
 &  $ME \downarrow$

Weight thro' centre does not pass thro'  $P$ ,  $\tau \neq 0$

(s)  $a \downarrow \Rightarrow F_x < Mg$   
 As  $Y \downarrow$ ,  $X$  (partly)  $\uparrow$ . so  $PE \uparrow$  Isolated system so  $ME = \text{constant}$   $\tau \neq 0$

(t)  $F_x = Mg$  since  $a = 0$ .  
 As  $Y \downarrow$ ,  $X$  (partly)  $\uparrow$ , so  $PE \uparrow$ . Due to viscous force,  $ME \downarrow$ .  $\tau \neq 0$

$\therefore$  (A)  $\rightarrow$  (p,t), (B)  $\rightarrow$  (q,s,t), (C)  $\rightarrow$  (p,r,t), (D)  $\rightarrow$  (q)

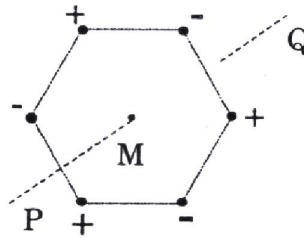
60. Six point charges, each of the same magnitude  $q$ , are arranged in different manners as shown in **Column II**. In each case, a point  $M$  and a line  $PQ$  passing through  $M$  are shown. Let  $E$  be the electric field and  $V$  be the electric potential at  $M$  (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line  $PQ$ . Let  $B$  be the magnetic field at  $M$  and  $\mu$  be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

**Column I**

**Column II**

(A)  $E = 0$

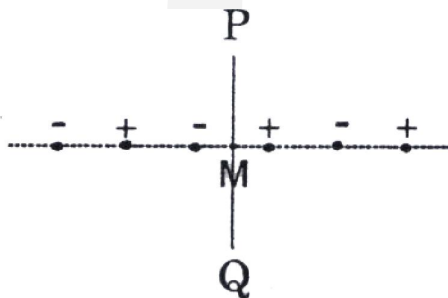
(p)



Charges are at the corners of a regular hexagon.  $M$  is at the centre of the hexagon.  $PQ$  is perpendicular to the plane of the hexagon.

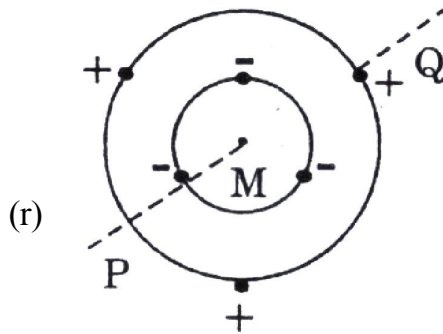
(B)  $V \neq 0$

(q)



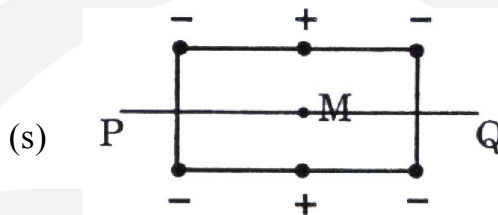
Charges are on a line perpendicular to  $PQ$  at equal intervals.  $M$  is the midpoint between the two innermost charges.

(C)  $B = 0$

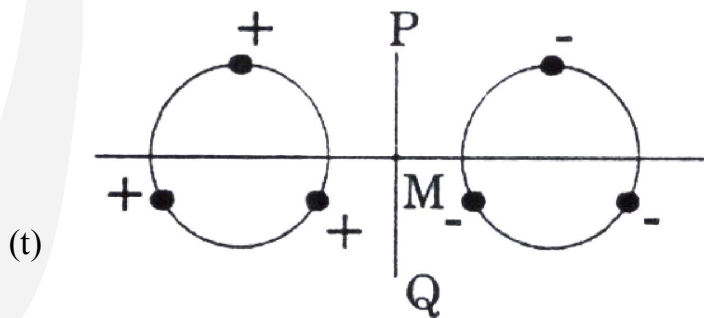


Charges are placed on two coplanar insulating rings at equal intervals  $M$  is the common centre of the rings.  $PQ$  is perpendicular to the plane of the rings.

(D)  $\mu \neq 0$



Charges are placed at the corners of a rectangle of sides  $a$  and  $2a$  and at the mid points of the longer sides.  $M$  is at the centre of the rectangle.  $PQ$  is parallel to the longer sides



Charges are placed on two coplanar, identical insulating rings at equal intervals.  $M$  is the mid-point between the centres of the rings.  $PQ$  is perpendicular to the line joining the centres and coplanar to the rings.

60. (P)  $E = 0, V = 0, B = 0, \mu = 0$   
 (Q)  $E \neq 0, V = 0, B = 0, \mu = 0$   
 (R)  $E = 0, V \neq 0, B \neq 0, \mu \neq 0$   
 (S)  $E = 0, V \neq 0, B \neq 0, \mu \neq 0$   
 (T)  $E \neq 0, V = 0, B = 0, \mu = 0$   
 $\therefore$  (A)  $\rightarrow$  (p,r,s), (B)  $\rightarrow$  (r,s),

(C)  $\rightarrow$  (p,q,t), (D)  $\rightarrow$  (r,s)