

IIT-JEE 2009

Answers by Aakash IIT-JEE

(Division of Aakash Educational Services Ltd.)

PAPER - I : CODES

Q.No.	-0	-1	-2	-3	-4	-5	-6	-7	-8	-9
01.	B	A	D	C	B	B	C	D	B	A
02.	B	A	B	B	A	A	B	B	B	C
03.	C	B	B	A	D	B	A	A	B	B
04.	A	D	C	B	B	B	B	C	D	B
05.	B	B	A	D	C	D	B	A	A	B
06.	A	B	B	A	B	C	D	B	A	D
07.	D	C	B	B	A	B	B	B	C	B
08.	B	B	A	B	B	A	A	B	B	A
09.	B,C	A,B	C,D	A,D	A,B	A,D	C,D	B,C	A,B	A,D
10.	C,D	A,D	B,C	A,B	C,D	B,C	A,B	A,D	B,C	A,B
11.	A,B	C,D	A,D	B,C	A,D	C,D	B,C	A,B	C,D	B,C
12.	A,D	B,C	A,B	C,D	B,C	A,B	A,D	C,D	A,D	C,D
13.	D	B	D	B	D	B	D	B	D	B
14.	C	A	C	A	C	A	C	A	C	A
15.	B	B	B	B	B	B	B	B	B	B
16.	B	D	B	D	B	D	B	D	B	D
17.	A	C	A	C	A	C	A	C	A	C
18.	B	B	B	B	B	B	B	B	B	B
19.	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p,q) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p,q) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p,q) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p,q) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p,q) D(r)
20.	A(p,q,s,t) B(p,s,t) C(p) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)	A(p,q,s,t) B(p,s,t) C(p) D(r)	A(p,r,t) B(s,t) C(p,q) D(p,q,s)
21.	A	A	D	C	D	C	C	D	B	C
22.	B	C	C	B	C	A	A	D	C	C
23.	C	D	A	A	D	B	C	A	A	D
24.	A	D	C	D	A	D	C	C	D	B
25.	D	A	A	D	C	D	B	C	A	C
26.	C	C	B	C	C	C	D	B	C	D
27.	D	C	D	A	A	A	D	C	C	A
28.	C	B	C	C	B	C	A	A	D	A
29.	B,C,D	B,C	A,C	A,B	B,C	A,B	A,C	B,C,D	B,C	A,B
30.	A,C	A,B	B,C,D	B,C	A,C	B,C,D	B,C	A,B	B,C,D	B,C
31.	B,C	A,C	A,B	B,C,D	A,B	A,C	B,C,D	B,C	A,C	B,C,D
32.	A,B	B,C,D	B,C	A,C	B,C,D	B,C	A,B	A,C	A,B	A,C
33.	A	A	A	A	A	A	A	A	A	A
34.	B	B	B	B	B	B	B	B	B	B
35.	B	D	B	D	B	D	B	D	B	D
36.	A	A	A	A	A	A	A	A	A	A
37.	B	B	B	B	B	B	B	B	B	B
38.	D	B	D	B	D	B	D	B	D	B
39.	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)
40.	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)	A(p) B(s,t) C(r) D(q,s)	A(p,q,s) B(p,t) C(p,q,r,t) D(s)
41.	B	A	A	C	C	D	C	A	B	D
42.	B	D	D	B	D	A	B	C	D	C
43.	C	C	B	A	A	B	D	A	B	C
44.	A	A	C	C	B	C	D	C	A	B
45.	C	B	A	A	C	A	B	D	A	D
46.	D	D	B	D	D	C	A	B	D	A
47.	A	C	C	B	A	B	C	D	C	B
48.	D	B	D	D	B	D	A	B	C	A
49.	A	A,D	C,D	B,D	A,D	B,D	C,D	A	A,D	B,D
50.	C,D	B,D	A	A,D	C,D	A	A,D	B,D	A	A,D
51.	A,D	C,D	B,D	A	B,D	C,D	A	A,D	C,D	A
52.	B,D	A	A,D	C,D	A	A,D	B,D	C,D	B,D	C,D
53.	D	A	D	A	D	A	D	A	D	A
54.	A	B	A	B	A	B	A	B	A	B
55.	B	D	B	D	B	D	B	D	B	D
56.	A	D	A	D	A	D	A	D	A	D
57.	B	A	B	A	B	A	B	A	B	A
58.	D	B	D	B	D	B	D	B	D	B
59.	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)
60.	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)	A(p,t) B(q,s,t) C(p,r,t) D(q)	A(p,r,s) B(r,s) C(p,q,t) D(r,s)

Though every care has been taken to provide the answers correctly but the Institute shall not be responsible for error, if any.

Aakash IIT-JEE

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Solutions to IIT-JEE 2009

Time : 3 hrs.

PAPER - I (Code - 0)

Max. Marks: 240

Instructions :

1. The question paper consists of 3 parts (Part I : Chemistry, Part II : Mathematics, Part III : Physics). Each part has 4 sections.
2. **Section I** contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which **only one is correct**.
3. **Section II** contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which **one or more is/are are correct**.
4. **Section III** contains 2 groups of questions. Each group has 3 questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct**.
5. **Section IV** contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with **one or more** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.
6. For each question in **Section I**, you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (-1) mark** will be awarded.
7. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded.
8. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded.
9. For each question in **Section IV**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is **no negative marking** for incorrect answer(s) for this section.

CHEMISTRY

PART- I

SECTION - I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

1. Given that the abundances of isotopes ^{54}Fe , ^{56}Fe and ^{57}Fe are 5%, 90% and 5%, respectively, the atomic mass of Fe is
- (A) 55.85 (B) 55.95 (C) 55.75 (D) 56.05

Answer (B)

Hints :

We will have to take weighted average

$$\frac{54 \times 5 + 90 \times 56 + 5 \times 57}{100} = 55.95$$

2. The term that corrects for the attractive forces present in a real gas in the van der Waals equation is
- (A) nb (B) $\frac{an^2}{V^2}$ (C) $-\frac{an^2}{V^2}$ (D) $-nb$

Answer (B)

Hints :

Correction factor in pressure is directly proportional to square of density

$$\therefore \text{It is } \frac{an^2}{V^2}$$

3. Among the electrolytes Na_2SO_4 , CaCl_2 , $\text{Al}_2(\text{SO}_4)_3$ and NH_4Cl , the most effective coagulating agent for Sb_2S_3 sol is
- (A) Na_2SO_4 (B) CaCl_2 (C) $\text{Al}_2(\text{SO}_4)_3$ (D) NH_4Cl

Answer (C)

Hints :

Coagulating power of an electrolyte is dependent on the valency of effective ion and Sb_2S_3 is negatively charged sol.

\therefore Answer is $\text{Al}_2(\text{SO}_4)_3$.

4. The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is
- (A) 4.0×10^{-4} (B) 4.0×10^{-5} (C) 5.0×10^{-4} (D) 4.0×10^{-6}

Answer (A)

Hints :

As the mole fraction of N_2 gas is 0.8 and total pressure is 5 atm. Therefore the partial pressure of N_2 will be 4 atm.

$$\text{Now } P_a = K_H H_a$$

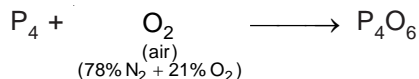
$$4 = 1 \times 10^5 \cdot \frac{n}{n+10} \quad (\because n \text{ is much less than } 10)$$

$$n = 4 \times 10^{-4} \text{ moles}$$

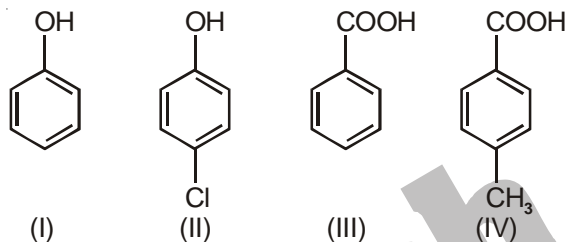
5. The reaction of P_4 with X leads selectively to P_4O_6 . The X is
- (A) Dry O_2 (B) A mixture of O_2 and N_2
 (C) Moist O_2 (D) O_2 in the presence of aqueous NaOH

Answer (B)

Hints :



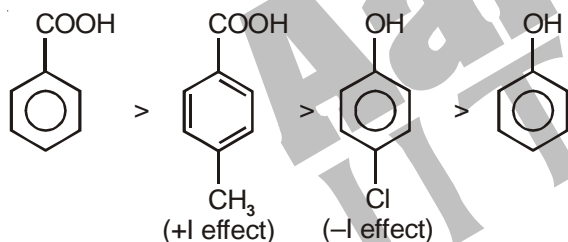
6. The correct acidity order of the following is



- (A) (III) > (IV) > (II) > (I) (B) (IV) > (III) > (I) > (II)
 (C) (III) > (II) > (I) > (IV) (D) (II) > (III) > (IV) > (I)

Answer (A)

Hints :



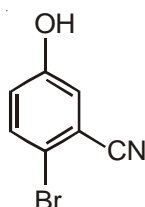
7. Among cellulose, poly(vinyl chloride), nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is
- (A) Nylon (B) Poly(vinyl chloride)
 (C) Cellulose (D) Natural Rubber

Answer (D)

Hints :

In natural rubber polar groups are absent and hence it will have only weak van der Waals force of attraction.

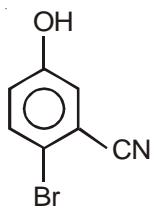
8. The IUPAC name of the following compound is



- (A) 4-Bromo-3-cyanophenol
 (B) 2-Bromo-5-hydroxybenzonitrile
 (C) 2-Cyano-4-hydroxybromobenzene
 (D) 6-Bromo-3-hydroxybenzonitrile

Answer (B)

Hints :



2-Bromo-5 hydroxy benzonitrile.

SECTION - II

Multiple Correct Answers Type

This section contains 4 multiple choice questions. Each Question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

9. The correct statement(s) regarding defects in solids is(are)
- (A) Frenkel defect is usually favoured by a very small difference in the sizes of cation and anion
 - (B) Frenkel defect is a dislocation defect
 - (C) Trapping of an electron in the lattice leads to the formation of F-center
 - (D) Schottky defects have no effect on the physical properties of solids

Answer (B, C)

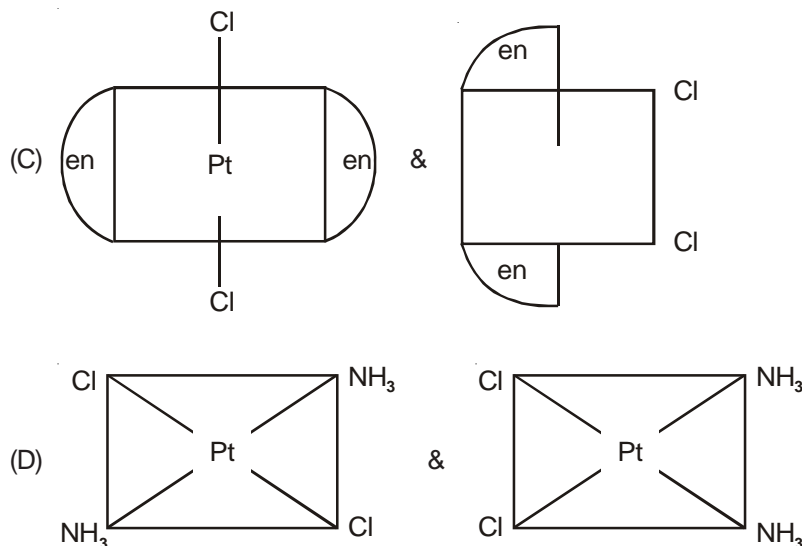
Hints :

- A → **Schottky** defect are favoured by small difference in sizes of cation and anion.
- B → In Frenkel defect cations are dislocated therefore. (True)
- C → Electron trapped in lattice are called F-centre. (True)
- D → Schottky defect **decrease density**.

10. The compound(s) that exhibit(s) geometrical isomerism is(are)
- (A) $[\text{Pt}(\text{en})\text{Cl}_2]$ (B) $[\text{Pt}(\text{en})_2]\text{Cl}_2$ (C) $[\text{Pt}(\text{en})_2\text{Cl}_2]\text{Cl}_2$ (D) $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$

Answer (C, D)

Hints :



11. The compound(s) formed upon combustion of sodium metal in excess air is(are)
- (A) Na_2O_2 (B) Na_2O (C) NaO_2 (D) NaOH

Answer (A, B)

12. The correct statement(s) about the compound $\text{H}_3\text{C}(\text{HO})\text{HC}=\text{CH}=\text{CH}(\text{OH})\text{CH}_3$ (**X**) is are
- (A) The total number of stereoisomers possible for **X** is 6
- (B) The total number of diastereomers possible for **X** is 3
- (C) If the stereochemistry about the double bond in **X** is *trans*, the number of enantiomers possible for **X** is 4
- (D) If the stereochemistry about the double bond in **X** is *cis*, the number of enantiomers possible for **X** is 2

Answer (A, D)

Hints :

Given compound has three stereocentre with symmetrical substituents. It will form six stereoisomers out of which 2 will be meso and one dl pair of *cis* and one dl pair of *trans*.

SECTION - III

Linked Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

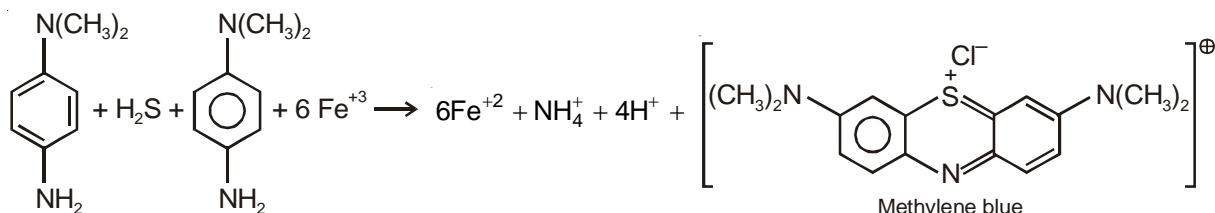
Paragraph for Question Nos. 13 to 15

p-Amino-*N,N*-dimethylaniline is added to a strongly acidic solution of **X**. The resulting solution is treated with a few drops of aqueous solution of **Y** to yield blue coloration due to the formation of methylene blue. Treatment of the aqueous solution of **Y** with the reagent potassium hexacyanoferrate(II) leads to the formation of an intense blue precipitate. The precipitate dissolves on excess addition of the reagent. Similarly, treatment of the solution of **Y** with the solution of potassium hexacyanoferrate (III) leads to a brown coloration due to the formation of **Z**.

13. The compound **X** is
- (A) NaNO_3 (B) NaCl
- (C) Na_2SO_4 (D) Na_2S

Answer (D)

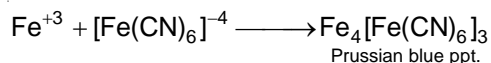
Hints :



14. The compound **Y** is
- (A) MgCl_2 (B) FeCl_2
- (C) FeCl_3 (D) ZnCl_2

Answer (C)

Hints :

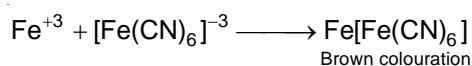


15. The compound **Z** is

- (A) $\text{Mg}_2[\text{Fe}(\text{CN})_6]$ (B) $\text{Fe}[\text{Fe}(\text{CN})_6]$
 (C) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$ (D) $\text{K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2$

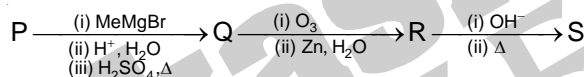
Answer (B)

Hints :



Paragraph for Question Nos. 16 to 18

A carbonyl compound **P**, which gives positive iodoform test, undergoes reaction with MeMgBr followed by dehydration to give an olefin **Q**. Ozonolysis of **Q** leads to a dicarbonyl compound **R**, which undergoes intramolecular aldol reaction to give predominantly **S**.

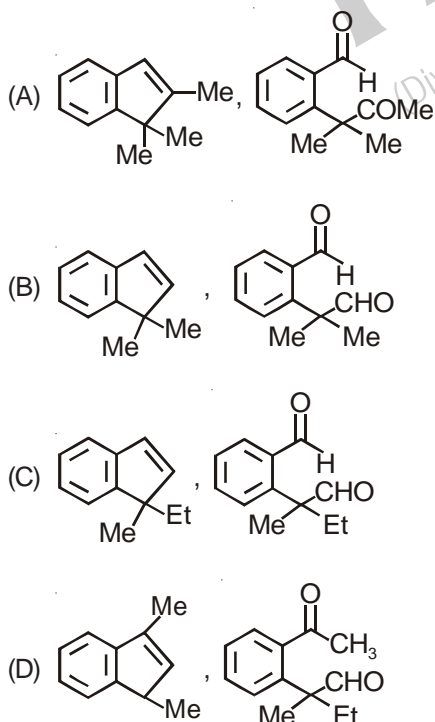


16. The structure of the carbonyl compound **P** is



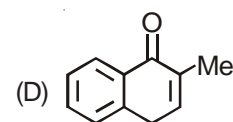
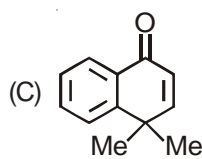
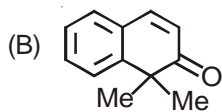
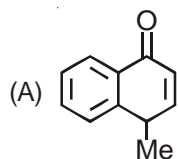
Answer (B)

17. The structures of the products **Q** and **R**, respectively, are



Answer (A)

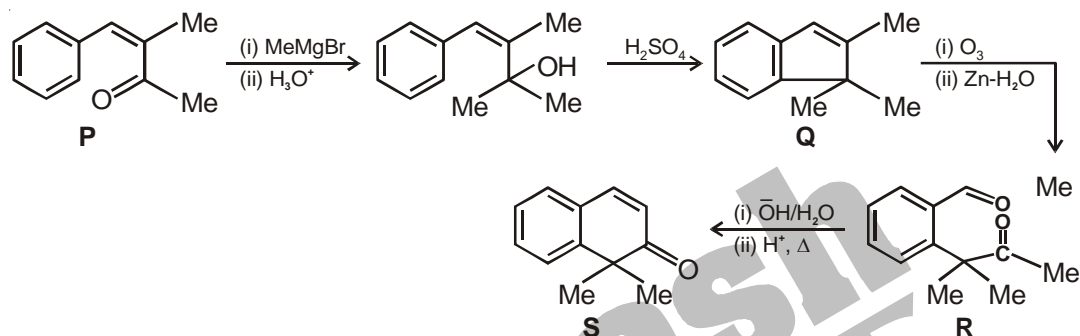
18. The structure of the product **S** is



Answer (B)

Hints :

Solution of Q.No. 16 to 18.



SECTION-IV

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **One OR More** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example

If the correct matches are A-p, s and t; B-q and r; C-p and q and D-s and t; then the correct darkening of bubbles will look like the following.

	p	q	r	s	t
A	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
B	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>

19. Match each of the diatomic molecules in **Column I** with its property/properties in **Column II**.

Column I

- (A) B₂
 (B) N₂
 (C) O₂⁻
 (D) O₂

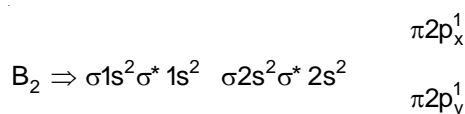
Column II

- (p) Paramagnetic
 (q) Undergoes oxidation
 (r) Undergoes reduction
 (s) Bond order ≥ 2
 (t) Mixing of 's' and 'p' orbitals

Answer A(p, r, t), B(s, t), C(p, q), D(p, q, s)

Hints :

Electronic configuration as



∴ It is paramagnetic, as highest occupied orbital is partially filled and it is of bonding nature, therefore it will readily undergo reduction.

When Z_{eff} is less like B and N, then energy difference between 2s and $2p_z$ is very small. Due to their same symmetry mixing of s and p orbital take place.

Similarly in N_2 b.o. is 3, it is diamagnetic, Neither it will undergo oxidation nor reduction.

O_2 and O_2^- are paramagnetic.

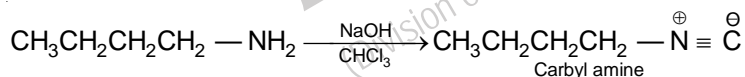
20. Match each of the compounds in **Column I** with its characteristic reaction(s) in.

Column I	Column II
(A) $CH_3CH_2CH_2CN$	(p) Reduction with Pd-C/ H_2
(B) $CH_3CH_2OCOCH_3$	(q) Reduction with $SnCl_2/HCl$
(C) $CH_3-CH=CH-CH_2OH$	(r) Development of foul smell on treatment with chloroform and alcoholic KOH
(D) $CH_3CH_2CH_2CH_2NH_2$	(s) Reduction with diisobutylaluminium hydride (DIBAL-H)
	(t) Alkaline hydrolysis

Answer A(p, q, s, t), B(p, s, t), C(p), D(r)

Hints :

- (A) $CH_3CH_2CH_2CN$ will give negative test with alkaline $CHCl_3$. It will give mentioned reaction with other reagents.
- (B) Esters will be reduced by catalytic hydrogenation, DIBAL-H and it will be hydrolyzed by alkaline hydrolysis.
- (C) Alkenes are only reduced by catalytic hydrogenation.
- (D) Primary amines will give +ve carbyl amine test with NaOH and $CHCl_3$.



MATHEMATICS

PART - II

SECTION - I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

21. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

Then the value of μ for which the vector \overline{PQ} is parallel to the plane $x - 4y + 3z = 1$ is

- | | |
|-------------------|--------------------|
| (A) $\frac{1}{4}$ | (B) $-\frac{1}{4}$ |
| (C) $\frac{1}{8}$ | (D) $-\frac{1}{8}$ |

Answer (A)

Hints :

$$\text{Let } \overline{OQ} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

$$\overline{OP} = (3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\overline{PQ} = \overline{OQ} - \overline{OP}$$

$$= ((\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})) - (3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$= (-2\hat{i} - 3\hat{j} - 4\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

$$\overline{PQ} = \hat{i}(-2 - 3\mu) + \hat{j}(-3 + \mu) + \hat{k}(-4 + 5\mu)$$

Vector perpendicular to plane is $\vec{n} = \hat{i} - 4\hat{j} + 3\hat{k}$

As \overline{PQ} is parallel to the plane

$$\Rightarrow \overline{PQ} \cdot \vec{n} = 0$$

$$\Rightarrow 1(-2 - 3\mu) - 4(-3 + \mu) + 3(-4 + 5\mu) = 0$$

$$\Rightarrow -2 - 3\mu + 12 - 4\mu - 12 + 15\mu = 0$$

$$8\mu - 2 = 0$$

$$\mu = \frac{1}{4}$$

22. Tangents drawn from the point $P(1, 8)$ to the circle

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

touch the circle at the point A and B . The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$

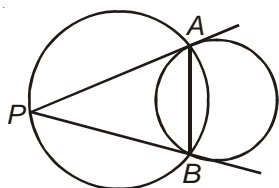
(B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$

(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Answer (B)

Hints :



The equation of AB is given by

$$x(1) + y(8) - 6\left(\frac{x+1}{2}\right) - 4\left(\frac{y+8}{2}\right) - 11 = 0$$

$$\Rightarrow x + 8y - 3(x + 1) - 2(y + 8) - 11 = 0$$

$$\Rightarrow -2x + 6y - 3 - 16 - 11 = 0$$

$$\Rightarrow -2x + 6y - 30 = 0$$

$$\Rightarrow 2x - 6y + 30 = 0$$

$$\Rightarrow x - 3y + 15 = 0$$

The equation of circum circle of ΔPAB may be written as

$$(x^2 + y^2 - 6x - 4y - 11) + \lambda(x - 3y + 15) = 0 \quad \dots(i)$$

The circle passes through $P(1, 8)$, hence

$$\Rightarrow (1 + 64 - 6 - 32 - 11) + \lambda(1 - 24 + 15) = 0$$

$$\Rightarrow (65 - 49) + \lambda(-23 + 15) = 0$$

$$16 + \lambda(-8) = 0$$

$$\lambda = 2$$

Consequently the equation of the circle is

$$x^2 + y^2 - 6x - 4y - 11 + 2(x - 3y + 15) = 0$$

$$x^2 + y^2 - 4x - 10y + 19 = 0$$

23. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$, and $f(0) = 0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer (C)

Hints :

We have

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, \quad 0 \leq x \leq 1$$

Differentiating both sides w.r.t. x we get

$$\sqrt{1 - (f'(x))^2} = f(x)$$

$$\Rightarrow 1 - [f'(x)]^2 = [f(x)]^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - y^2 \quad \left[\text{put } y = f(x) \text{ \& } \frac{dy}{dx} = f'(x) \right]$$

$$\Rightarrow \int \frac{1}{\sqrt{1 - y^2}} dy = \pm \int 1 dx$$

$$\Rightarrow \sin^{-1} y = c \pm x$$

$$\Rightarrow y = \sin(c \pm x)$$

$$\Rightarrow f(x) = \sin(c \pm x)$$

$$\Rightarrow f(0) = \sin c \quad [f(0) = 0]$$

$$\Rightarrow 0 = \sin c$$

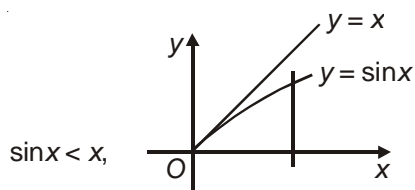
$$\Rightarrow c = 0$$

$$\text{Hence } f(x) = \sin(\pm x) = \pm \sin x$$

But $f(x) = -\sin x$ is rejected as f is given to be non-negative

Hence $y = \sin x$

For $x \in (0, 1)$



$$\therefore \sin \frac{1}{2} < \frac{1}{2} \text{ \& \; } \sin \frac{1}{3} < \frac{1}{3}$$

24. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

- (A) 48 (B) 32
(C) 40 (D) 80

Answer (A)

Hints :

$$z\bar{z}^3 + \bar{z}z^3 = 350$$

$$|z|^2 (\bar{z}^2) + |z|^2 (z^2) = 350$$

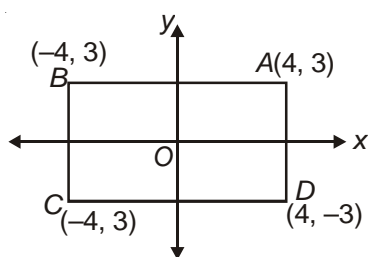
$$|z|^2 (z^2 + \bar{z}^2) = 350$$

$$\Rightarrow (x^2 + y^2)(x^2 - y^2 + 2ixy + x^2 - y^2 - 2ixy) = 350$$

$$\Rightarrow 2(x^2 + y^2)(x^2 - y^2) = 350$$

$$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175 = (3^2 + 4^2)(4^2 - 3^2)$$

Which suggests that points (x, y) satisfying the given equation are $(4, 3), (-4, -3), (-4, 3), (4, -3)$



Required area = $AB \times BC$

$$= 8 \times 6 = 48 \text{ sq.units}$$

25. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M . Then the area of the triangle with vertices at A, M and the origin O is

- (A) $\frac{31}{10}$ (B) $\frac{29}{10}$
(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Answer (D)**Hints :**

The given ellipse is

$$x^2 + 9y^2 = 9 \Rightarrow \frac{x^2}{9} + \frac{y^2}{1} = 1$$

Now, equation of auxiliary circle is $x^2 + y^2 = 9$

AM is chord of circle, $x^2 + y^2 = 9$

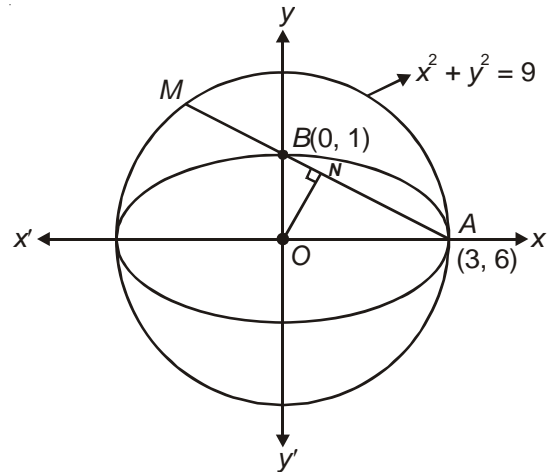
The equation of AM is $\frac{x}{3} + \frac{y}{1} = 1 \Rightarrow x + 3y = 3$

Now, perpendicular distance $ON = \left| \frac{3}{\sqrt{10}} \right|$

$$\begin{aligned} NA &= \sqrt{9 - \frac{9}{10}} \\ &= \sqrt{\frac{81}{10}} = \frac{9}{\sqrt{10}} \end{aligned}$$

$$AM = 2 \times \frac{9}{\sqrt{10}}$$

$$\text{Required area of } \Delta AOM = \frac{1}{2} \times 2 \times \frac{9}{\sqrt{10}} \times \frac{3}{\sqrt{10}} = \frac{27}{10}$$



26. If $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then

- (A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
 (C) \vec{b}, \vec{d} are non-parallel (D) \vec{a}, \vec{d} are parallel and \vec{b}, \vec{c} are parallel

Answer (C)**Hints :**

Let the angle between vectors \hat{a} and \hat{b} , \hat{b} and \hat{c} , \hat{c} and \hat{d} be α, β and γ respectively

$$\Rightarrow (\hat{a} \times \hat{b}) \cdot (\hat{c} \times \hat{d}) = 1$$

$$(\sin \alpha \hat{n}_1) \cdot (\sin \beta \hat{n}_2) = 1$$

$$\Rightarrow \sin \alpha \cdot \sin \beta \cdot (\hat{n}_1 \cdot \hat{n}_2) = 1$$

Let the angle between \hat{n}_1 & \hat{n}_2 be θ

$$\Rightarrow \sin \alpha \sin \beta \cos \theta = 1$$

This is possible when

$$\sin \alpha = \sin \beta = \cos \theta = 1 \quad \dots(i)$$

$$\text{Also, } \vec{a} \cdot \vec{c} = \frac{1}{2}$$

$$\Rightarrow 1 \times 1 \times \cos \gamma = \frac{1}{2}$$

$$\Rightarrow \gamma = \frac{\pi}{3} \quad \dots(ii)$$

From (i), we have

$$\alpha = \beta = \frac{\pi}{2}$$

$$\theta = 0^\circ$$

As $\theta = 90^\circ$, then \hat{a} , \hat{b} , \hat{c} , \hat{d} will be coplanar. Option (A) and (B) are wrong. Option (D) also is not true. Hence option (C) is correct.

27. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2^\circ$ is

(A) $\frac{1}{\sin 2^\circ}$

(B) $\frac{1}{3\sin 2^\circ}$

(C) $\frac{1}{2\sin 2^\circ}$

(D) $\frac{1}{4\sin 2^\circ}$

Answer (D)

Hints : $z = \cos\theta + i\sin\theta = e^{i\theta}$

Now, $\sum_{m=1}^{15} \text{Im} e^{i(2m-1)\theta}$

$$= \sin\theta + \sin 3\theta + \dots + \sin 29\theta$$

$$= \frac{\sin \frac{15 \cdot 2\theta}{2}}{\sin \left(\frac{2\theta}{2}\right)} \cdot \sin \left(\frac{2\theta + (15-1) \times 2\theta}{2}\right)$$

$$= \frac{\sin 15\theta \cdot \sin 15\theta}{\sin \theta}$$

For $\theta = 2^\circ$, the given expression reduces to

$$= \frac{\sin 30^\circ \cdot \sin 30^\circ}{\sin 2^\circ} = \frac{1}{4\sin 2^\circ}$$

28. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2, and 3 only, is

(A) 55

(B) 66

(C) 77

(D) 88

Answer (C)

Hints :

Since sum of digits of seven digit numbers formed by the digits 1, 2 and 3 is 10, hence following cases may arise :

Case I :

$$1 + 1 + 1 + 1 + 1 + 2 + 3 = 10$$

$$\text{Number of possible seven digit numbers} = \frac{7!}{5!} = 42$$

Case II :

$$1 + 1 + 1 + 1 + 2 + 2 + 2 = 10$$

$$\text{Number of possible seven digit numbers} = \frac{7!}{3!4!} = 35$$

$$\text{Total number of seven digit numbers} = 42 + 35 = 77$$

SECTION - II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which **ONE OR MORE** is/are correct.

29. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(A) $e - 1$

(B) $\int_1^e \ln(e+1-y) dy$

(C) $e - \int_0^1 e^x dx$

(D) $\int_1^e \ln y dy$

Answer (B, C, D)

Hints :

The common region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is shown in the adjoining figure

Required area = $\int_0^1 (e - e^x) dx$

$$e - \int_0^1 e^x dx$$

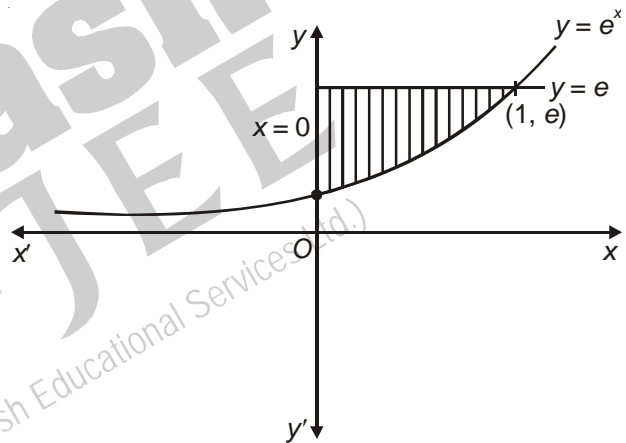
$$= e - e + 1 = 1$$

Also $\int_1^e \ln y dy$

$$[y \ln y - y]_1^e$$

$$= [e - e + 1] = 1$$

Also $\int_1^e \ln y dy = \int_1^e \ln(e+1-y) dy$



30. Let

$$L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$$

If L is finite, then

(A) $a = 2$

(B) $a = 1$

(C) $L = \frac{1}{64}$

(D) $L = \frac{1}{32}$

Answer (A, C)

Hints :

We have

$$L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$$

For L to be finite

$$= \lim_{x \rightarrow 0} \frac{\frac{(-2x)}{2\sqrt{a^2 - x^2}} - \frac{2x}{4}}{4x^3} \quad (\text{using L' hospital rule})$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x}{\sqrt{a^2 - x^2}} - \frac{x}{2}}{4x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{a^2 - x^2}} - \frac{1}{2}}{4x^2}$$

For L to be real, $a = 2$

Also, again using L' hospital rule

$$\lim_{x \rightarrow 0} \frac{-\frac{1}{2}(a^2 - x^2)^{-3/2} \cdot (-2x)}{8x}$$

$$L = \frac{-\frac{1}{2}(2^2 - 0)^{-3/2}(-2)}{8} = \frac{1}{64}$$

31. In a triangle ABC with fixed base BC , the vertex A moves such that

$$\cos B + \cos C = 4 \sin^2 \frac{A}{2}$$

If a , b and c denote the lengths of the sides of the triangle opposite to the angles A , B and C respectively, then

(A) $b + c = 4a$

(B) $b + c = 2a$

(C) Locus of point A is an ellipse

(D) Locus of point A is a pair of straight lines

Answer (B, C)

Hints : Given $\cos B + \cos C = 4 \sin^2 \frac{A}{2}$

$$\Rightarrow 2 \cos\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) = 4 \sin^2 \frac{A}{2}$$

$$\Rightarrow 2 \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) = 4 \sin^2 \frac{A}{2}$$

$$\Rightarrow \cos\left(\frac{B-C}{2}\right) = 2 \sin \frac{A}{2}, \text{ as } \sin \frac{A}{2} \neq 0$$

$$\Rightarrow \frac{\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)} = 2$$

$$\Rightarrow \frac{1 + \tan \frac{B}{2} \cdot \tan \frac{C}{2}}{1 - \tan \frac{B}{2} \cdot \tan \frac{C}{2}} = 2$$

$$\Rightarrow \frac{1 + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \cdot \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}}{1 - \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \cdot \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}} = 2$$

$$\Rightarrow \frac{1 + \frac{(s-a)}{s}}{1 - \frac{s-a}{s}} = 2$$

$$\Rightarrow \frac{2s-a}{a} = 2$$

$$\Rightarrow \frac{b+c}{a} = 2$$

$$\boxed{b+c=2a}$$

and also $b+c=2a$

$$\Rightarrow AC + AB = 2BC$$

$\Rightarrow AC + AB > BC$, which shows that the locus of point A is an ellipse.

32. If

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5},$$

then

(A) $\tan^2 x = \frac{2}{3}$

(C) $\tan^2 x = \frac{1}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

Answer (A, B)

Hints :

We have,

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$

$$\Rightarrow 3 \sin^4 x + 2 \cos^4 x = \frac{6}{5}$$

$$\Rightarrow 3(1 - \cos^2 x)^2 + 2 \cos^4 x = \frac{6}{5}$$

$$\Rightarrow 3(1 + \cos^4 x - 2 \cos^2 x) + 2 \cos^4 x = \frac{6}{5}$$

$$\Rightarrow 3 + 3 \cos^4 x - 6 \cos^2 x + 2 \cos^4 x = \frac{6}{5}$$

$$\Rightarrow 5 \cos^4 x - 6 \cos^2 x + 3 - \frac{6}{5} = 0$$

$$\Rightarrow 5\cos^4 x - 6\cos^2 x + \frac{9}{5} = 0$$

$$\Rightarrow 25\cos^4 x - 30\cos^2 x + 9 = 0$$

$$\Rightarrow (5\cos^2 x - 3)^2 = 0$$

$$\Rightarrow \cos^2 x = \frac{3}{5}$$

$$\Rightarrow \sec^2 x = \frac{5}{3}$$

$$\tan^2 x = \sec^2 x - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

Hence option 'A' is true

$$\sin^2 x = 1 - \cos^2 x = 1 - \frac{3}{5} = \frac{2}{5}$$

Now, $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27}$

$$= \frac{\left(\frac{2}{5}\right)^4}{8} + \frac{\left(\frac{3}{5}\right)^4}{27} = \frac{2}{5^4} + \frac{3}{5^4} = \frac{5}{5^4} = \frac{1}{125} \text{ option (B)}$$

SECTION - III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 33 to 35

Let \mathbb{A} be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

33. The number of matrices in \mathbb{A} is

- (A) 12 (B) 6
(C) 9 (D) 3

Answer (A)

Hints :

The matrix is of order 3×3 where there are 9 elements. This has to be symmetric matrix with five of the entries as 1 and four of the entries as zero. For the matrix to be symmetric (i) either all the three elements along principal diagonal should be (1) or exactly one element along principal diagonal should be 1.

The possible matrixes are as follows

$$A_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, C_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, D_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

There are 12 such matrices.

34. The number of matrices A in \mathbb{A} for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

- (A) Less than 4 (B) At least 4 but less than 7
(C) At least 7 but less than 10 (D) At least 10

Answer (B)

Hints :

The determinant corresponding to matrices B_2, B_3, C_1, C_3, D_1 and D_2 are non-zero. Therefore in these three cases, the given linear equations will have unique solution. Number of required matrices in this case is 6.

35. The number of matrices A in \mathbb{A} for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

- (A) 0 (B) More than 2
(C) 2 (D) 1

Answer (B)

Hints :

(i) If we take (i) $A_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

then $|A_1| = 0$

$$\Delta x = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0, \text{ hence in this case system is inconsistent.}$$

(ii) If we take $A_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow |A_2| = 0$

$$\Delta x = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0, \text{ hence system is inconsistent.}$$

(iii) If we take $A_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow |A_3| = 0$

$$\Delta x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0, \text{ Similarly } \Delta y = \Delta z = 0. \text{ In this case we get infinite solution.}$$

(iv) If we take $B_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} \Rightarrow |B_1| = 0. \text{ In this case } \Delta x = \Delta y = \Delta z = 0$

Hence infinite solution consistent system.

Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

36. The probability that $X = 3$, equals

(A) $\frac{25}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{125}{216}$

Answer (A)

Hints : Clearly $p = \frac{1}{6}, q = \frac{5}{6}$

$$p(X = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

37. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{25}{216}$

Answer (B)

Hints : We have

$$p(X \geq 3) = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \dots$$

$$= \frac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}} = \frac{25}{36}$$

38. The conditional probability that $X \geq 6$ given $X = 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{216}$

(C) $\frac{5}{36}$

(D) $\frac{25}{36}$

Answer (D)

Hints : The conditional probability that $X \geq 6$
given $X > 3$

$$\frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^7 \cdot \frac{1}{6} + \dots}{\left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \dots}$$

$$= \frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} \left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots\right)}{\left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} \left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots\right)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

SECTION - IV

Matrix - Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A - p, s and t; B - q and r; C - p and q; and D - s and t; then the correct darkening of bubbles will look like the following.

	p	q	r	s	t
A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
C	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
D	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>

39. Match the statements/expressions in **Column I** with the open intervals in **Column II**.

Column I

Column II

- (A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$ (p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$ (q) $\left(0, \frac{\pi}{2}\right)$
- (C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies (r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
- (D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing (s) $\left(0, \frac{\pi}{8}\right)$
- (t) $(-\pi, \pi)$

Answer A(p, q, s); B(p, t); C(p, q, r, t); D(s)

Hints :

We have,

$$(A) \frac{y'}{y} = -\frac{1}{(x-3)^2}$$

$$\Rightarrow \int \frac{dy}{y} = \int -\frac{dx}{(x-3)^2}$$

$$\Rightarrow \log |y| = \frac{1}{x-3} + c$$

$$\Rightarrow x \neq 3$$

$$(B) I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$$

$$I = \int_1^5 (6-x-1)(6-x-2)(6-x-3)(6-x-4)(6-x-5)dx$$

$$I = \int_1^5 (5-x)(4-x)(3-x)(2-x)(1-x)dx$$

$$I = \int_1^5 -(x-5)(x-4)(x-3)(x-2)(x-1)dx$$

$$\Rightarrow I = -I$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

(C) $y = 1 - \sin^2 x + \sin x$

$$y = 1 - \left(\sin^2 x - \sin x + \frac{1}{4} - \frac{1}{4} \right)$$

$$y = \frac{5}{4} - \left(\sin x - \frac{1}{2} \right)^2$$

y is maximum at $\sin x = \frac{1}{2}$

$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\Rightarrow x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \cup \left(0, \frac{\pi}{2} \right) \cup (-\pi, \pi) \cup \left(\frac{\pi}{8}, \frac{5\pi}{4} \right)$$

(D) $y = \tan^{-1}(\sin x + \cos x)$

$$\frac{dy}{dx} = \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$$

For increasing function $\frac{dy}{dx} > 0$

$$\Rightarrow \cos x - \sin x > 0$$

$$\Rightarrow \cos x > \sin x$$

$$\Rightarrow x \in \left(0, \frac{\pi}{8} \right)$$

40. Match the conics in **Column I** with the statements/expressions in **Column II**.

Column I

(A) Circle

(B) Parabola

(C) Ellipse

(D) Hyperbola

Column II

(p) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$

(q) Points z in the complex plane satisfying $|z + 2| - |z - 2| = \pm 3$

(r) Points of the conic have parametric representation

$$x = \sqrt{3} \left(\frac{1-t^2}{1+t^2} \right), y = \frac{2t}{1+t^2}$$

(s) The eccentricity of the conic lies in the interval $1 \leq e < \infty$

(t) Points z in the complex plane satisfying $\operatorname{Re}(z + 1)^2 = |z|^2 + 1$

Answer A(p); B(s, t); C(r); D(q, s)

Hints :

(p) The line $hx + ky = 1$ touches $x^2 + y^2 = 4$

$$\text{Hence } \frac{1}{\sqrt{h^2 + k^2}} = 2$$

$$\Rightarrow h^2 + k^2 = \frac{1}{4}$$

\Rightarrow Locus of (h, k) is $x^2 + y^2 = \frac{1}{4}$, which is a circle.

(q) We have

$$|z + 2| - |z - 2| = \pm 3$$

Which shows that the difference of distances of the moving point from $(-2, 0)$ and $(2, 0)$ is 3 (less than 4),

Hence locus of the point (z) is a hyperbola.

(r) $\frac{x}{\sqrt{3}} = \frac{1-t^2}{1+t^2}$

$$y = \frac{2t}{1+t^2}$$

Squaring and adding we get

$$\frac{x^2}{3} + \frac{y^2}{1} = \frac{(1-t^2)^2 + 4t^2}{(1+t^2)^2} = 1$$

which represents an ellipse.

(s) When eccentricity e lies in

$$1 \leq e < \infty$$

For $e = 1$, conic is parabola

For $1 < e < \infty$, conic is hyperbola

(t) We have

$$\operatorname{Re}(z + 1)^2 = |z|^2 + 1$$

$$\Rightarrow (x + 1)^2 = x^2 + y^2 + 1$$

$$\Rightarrow x^2 + 2x + 1 = x^2 + y^2 + 1$$

$$\Rightarrow y^2 = 2x$$

which represents a parabola.

PHYSICS

PART - III

SECTION - I

Single Correct Choice Type

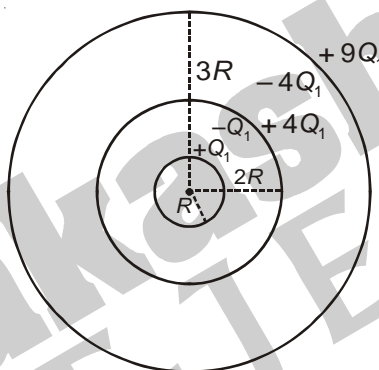
This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

41. Three concentric metallic spherical shells of radii $R, 2R, 3R$, are given charges Q_1, Q_2, Q_3 , respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

- (A) 1 : 2 : 3 (B) 1 : 3 : 5 (C) 1 : 4 : 9 (D) 1 : 8 : 18

Answer (B)

Hints : Charge distribution will be as shown.



$$Q_2 = 4Q_1 - Q_1 = 3Q_1$$

$$Q_3 = 9Q_1 - 4Q_1 = 5Q_1$$

$$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$$

42. A block of base $10 \text{ cm} \times 10 \text{ cm}$ and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0° . Then

- (A) At $\theta = 3^\circ$, the block will start sliding down the plane
(B) The block will remain at rest on the plane up to certain θ and then it will topple
(C) At $\theta = 60^\circ$, the block will start sliding down the plane and continue to do so at higher angles
(D) At $\theta = 60^\circ$, the block will start sliding down the plane and on further increasing θ , it will topple at certain θ

Answer (B)

Hints :

Block will topple if $\tan\theta > \frac{a}{h}$

$$\text{i.e., } \tan\theta > \frac{2}{3} \text{ or } \theta > \tan^{-1}\left(\frac{2}{3}\right)$$

Block will slide if $\tan\theta > \mu$

$$\text{i.e., } \tan\theta > \sqrt{3} \text{ or } \theta > 60^\circ$$

hence the block will topple before sliding.

43. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take $g = 10 \text{ m/s}^2$.]
- (A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

Answer (C)

Hints :

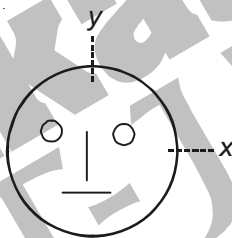
$$\text{Speed of ball} = \sqrt{2 \times 10 \times 7.2} = 12 \text{ m/s}$$

$$\text{speed of ball as seen by fish} = \mu v$$

$$= \frac{4}{3} \times 12$$

$$= 16 \text{ m/s}$$

44. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m . The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are : outer circle $(0, 0)$, left inner circle $(-a, a)$, right inner circle (a, a) , vertical line $(0, 0)$ and horizontal line $(0, -a)$. The y -coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$

(B) $\frac{a}{8}$

(C) $\frac{a}{12}$

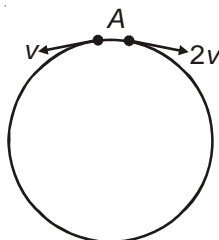
(D) $\frac{a}{3}$

Answer (A)

Hints :

$$y_{\text{cm}} = \frac{2ma - ma}{10m} = \frac{a}{10}$$

45. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(A) 4

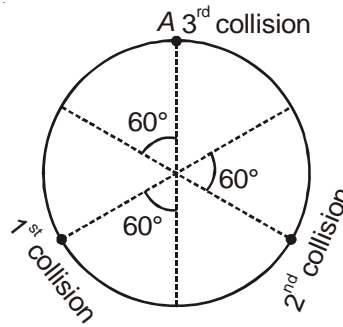
(B) 3

(C) 2

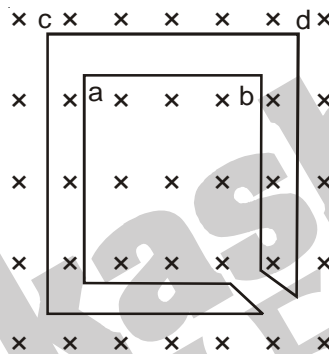
(D) 1

Answer (C)

Hints :



46. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then



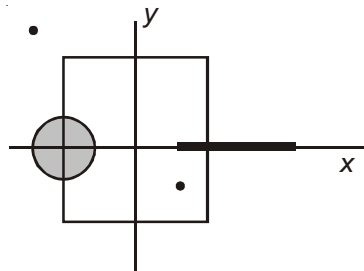
- (A) $I_1 > I_2$
 (B) $I_1 < I_2$
 (C) I_1 is in the direction **ba** and I_2 is in the direction **cd**
 (D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

Answer (D)

Hints :

According to Lenz law I_1 is from **a** to **b** and I_2 from **c** to **d**.

47. A disk of radius $a/4$ having a uniformly distributed charge 6 C is placed in the x - y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8 C is placed on the x -axis from $x = a/4$ to $x = 5a/4$. Two point charges -7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is



- (A) $\frac{-2C}{\epsilon_0}$ (B) $\frac{2C}{\epsilon_0}$ (C) $\frac{10C}{\epsilon_0}$ (D) $\frac{12C}{\epsilon_0}$

Answer (A)

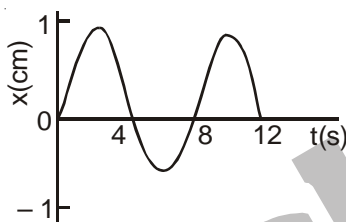
Hints :

$$q_{in} = \underset{\text{(Disc)}}{3C} + \underset{\text{(Rod)}}{2C} + \underset{\text{(Point charge)}}{(-7C)}$$

$$q_{in} = -2C$$

$$\text{Flux} = \frac{-2C}{\epsilon_0}$$

48. The $x-t$ graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is



- (A) $\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$ (B) $-\frac{\pi^2}{32} \text{ cm/s}^2$ (C) $\frac{\pi^2}{32} \text{ cm/s}^2$ (D) $-\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$

Answer (D)

Hints :

$$a = -\omega^2 A \sin \omega t$$

$$\Rightarrow a = -\frac{\pi^2}{16} \times 1 \times \sin \frac{2\pi}{8} \times \frac{4}{3}$$

$$\Rightarrow a = -\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$$

SECTION - II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each Question has 4 choices (A), (B), (C) and (D), for its answer, out of which **ONE OR MORE** is/are correct

49. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that
- (A) Linear momentum of the system does not change in time
 - (B) Kinetic energy of the system does not change in time
 - (C) Angular momentum of the system does not change in time
 - (D) Potential energy of the system does not change in time

Answer (A)

Hints :

According to law of conservation of linear momentum if $\vec{F} = 0$ linear momentum of the system does not change in time.

50. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded, is (are)

- (A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

Answer (C, D)

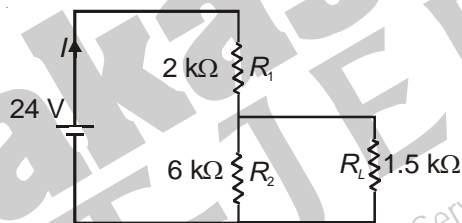
Hints : For option (A) $v = \frac{uf}{u-f} = 56$ cm

For option (B) $v = \frac{uf}{u-f} = 48$ cm

For option (C) $v = \frac{uf}{u-f} = 37.7$ cm

For option (D) $v = \frac{uf}{u-f} = 34.66$ cm

51. For the circuit shown in the figure



- (A) The current I through the battery is 7.5 mA
 (B) The potential difference across R_L is 18 V
 (C) Ratio of powers dissipated in R_1 and R_2 is 3
 (D) If R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

Answer (A, D)

Hints : For mesh 1,

$$2000I + 6000I_2 = 24$$

$$I_1 + 4I_2 = 0.012 \quad \dots (i)$$

For mesh 2

$$1500I_1 - 6000I_2 = 0$$

$$I_1 = 4I_2 \quad \dots (ii)$$

From (i) and (ii)

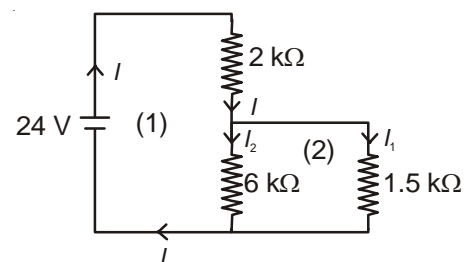
$$I_2 = 1.5 \text{ mA}$$

$$I_1 = 6 \text{ mA}$$

$$I = 7.5 \text{ mA}$$

P.D. across $R_L = 6 \times 10^{-3} \times 1500 = 9$ volt

$$\frac{(P)_{R_1}}{(P)_{R_2}} = \frac{(7.5)^2 \times 2}{(1.5)^2 \times 6} = \frac{25}{3}$$



$$(P)_{R_L} = 36 \times 10^{-6} \times 1500 = 54 \text{ mW}$$

If R_1 and R_2 are interchanged

$$I \times 6000 + I_2 \times 200 = 24$$

$$3I_1 + 4I_2 = 0.012$$

$$1500I_1 - 2000I_2 = 0$$

$$3I_1 = 4I_2$$

$$I_2 = 1.5 \text{ mA}$$

$$I_1 = 2 \text{ mA}$$

$$(P)_{R_L} = 4 \times 10^{-6} \times 1500 = 6 \text{ mW}$$

$$\frac{\text{Power dissipated in } R_L \text{ finally}}{\text{Power dissipated in } R_L \text{ Initially}} = \frac{6}{54} = \frac{1}{9}$$

52. C_V and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then
- (A) $C_p - C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
- (B) $C_p + C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
- (C) $\frac{C_p}{C_V}$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
- (D) $C_p \cdot C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Answer (B, D)

Hints : $(C_p + C_V)_{\text{Diatomic}} = \frac{7R}{2} + \frac{5R}{2} = 6R$

$$(C_p + C_V)_{\text{Monoatomic}} = \frac{5R}{2} + \frac{3R}{2} = 4R$$

$$(C_p \cdot C_V)_{\text{Diatomic}} = \frac{35R^2}{4}$$

$$(C_p \cdot C_V)_{\text{Monoatomic}} = \frac{15R^2}{4}$$

$$\frac{C_p}{C_V} = \gamma = 1 + \frac{2}{f} \text{ is smaller for diatomic gas}$$

$$C_p - C_V = R = \text{constant}$$

SECTION - III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct

Paragraph for Question Nos. 53 to 55

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1\text{H}$, known as deuteron and denoted by D , can be thought of as a candidate for fusion reactor. The $D-D$ reaction is ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than $5 \times 10^{14} \text{ s/cm}^2$.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5} \text{ eV/K}$; $\frac{e^2}{4\pi\epsilon_0} = 1.44 \times 10^{-19} \text{ eVm}$

53. In the core of nuclear fusion reactor, the gas becomes plasma because of

- (A) Strong nuclear force acting between the deuterons
- (B) Coulomb force acting between the deuterons
- (C) Coulomb force acting between deuteron-electron pairs
- (D) The high temperature maintained inside the reactor core

Answer (D)

Hints : Electrons get detached from nucleus as they acquire ionisation energy at high temperature.

54. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5 kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of $4 \times 10^{-15} \text{ m}$ is in the range

- (A) $1.0 \times 10^9 \text{ K} < T < 2.0 \times 10^9 \text{ K}$
- (B) $2.0 \times 10^9 \text{ K} < T < 3.0 \times 10^9 \text{ K}$
- (C) $3.0 \times 10^9 \text{ K} < T < 4.0 \times 10^9 \text{ K}$
- (D) $4.0 \times 10^9 \text{ K} < T < 5.0 \times 10^9 \text{ K}$

Answer (A)

Hints : $\frac{e^2}{4\pi\epsilon_0 r} = 3kT$

$$T = 1.4 \times 10^9 \text{ K}$$

55. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?
- (A) Deuteron density = $2.0 \times 10^{12} \text{ cm}^{-3}$, confinement time = $5.0 \times 10^{-3} \text{ s}$
 (B) Deuteron density = $8.0 \times 10^{14} \text{ cm}^{-3}$, confinement time = $9.0 \times 10^{-1} \text{ s}$
 (C) Deuteron density = $4.0 \times 10^{23} \text{ cm}^{-3}$, confinement time = $1.0 \times 10^{-11} \text{ s}$
 (D) Deuteron density = $1.0 \times 10^{24} \text{ cm}^{-3}$, confinement time = $4.0 \times 10^{-12} \text{ s}$

Answer (B)

Hints : $nt_0 > 5 \times 10^{14}$

Paragraph for Question Nos. 56 to 58

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation.

The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number ' n ' taking values 1, 2, 3,... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ and $x = a$. Take $h = 6.6 \times 10^{-34} \text{ J s}$ and $e = 1.6 \times 10^{-19} \text{ C}$.

56. The allowed energy for the particle for a particular value of n is proportional to
- (A) a^{-2} (B) $a^{-3/2}$
 (C) a^{-1} (D) a^2

Answer (A)

Hints : $n \frac{\lambda}{2} = a \Rightarrow \lambda = \frac{2a}{n} \Rightarrow p = \frac{nh}{2a}$

As $E = \frac{p^2}{2m} \Rightarrow E \propto a^{-2}$

57. If the mass of the particle is $m = 1.0 \times 10^{-30} \text{ kg}$ and $a = 6.6 \text{ nm}$, the energy of the particle in its ground state is closest to
- (A) 0.8 meV (B) 8 meV
 (C) 80 meV (D) 800 meV

Answer (B)

Hints:

$$E = \frac{n^2 h^2}{8a^2 m}$$

$$E = 7.8 \text{ MeV}$$

58. The speed of the particle, that can take discrete values, is proportional to
- (A) $n^{-3/2}$ (B) n^{-1}
 (C) $n^{1/2}$ (D) n

Answer (D)

Hints:

$$p = \frac{nh}{2a}$$

$$\Rightarrow v = \frac{nh}{2am}$$

SECTION-IV

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example.

If the correct matches are A-p, s and t ; B-q and r ; C-p and q; and D-s and t; then the correct darkening of bubbles will look like the following.

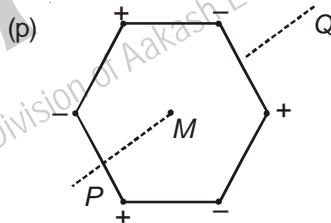
	p	q	r	s	t
A	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
B	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>

59. Six point charges each of the same magnitude q , are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ . Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current

Column I

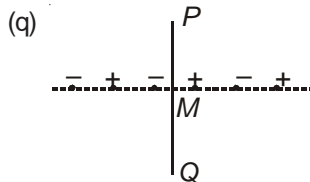
Column II

(A) $E = 0$



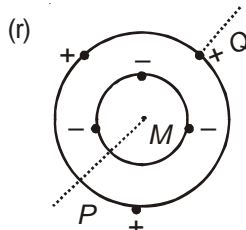
Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon

(B) $V \neq 0$



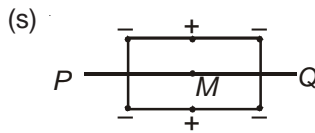
Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges

(C) $B = 0$

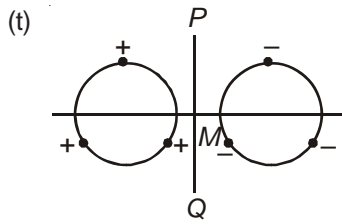


Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(D) $\mu \neq 0$



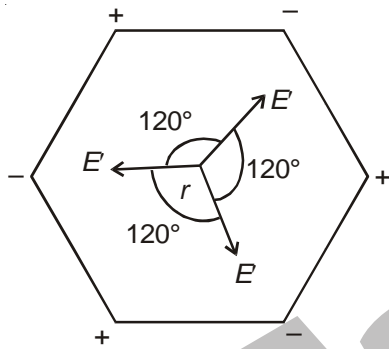
Charges are placed at the two corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides M is at the centre of the rectangle. PQ is parallel to the longer sides



Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings

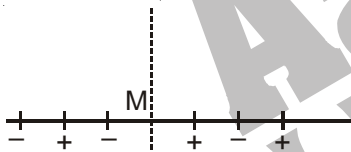
Answer A(p,r,s), B(r,s), C(p,q,t), D(r, s)

Hints:



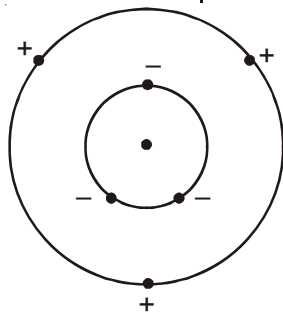
$E_{\text{net}} = 0, V = 0$

On rotating net current is zero, hence $B = 0, \mu = 0$



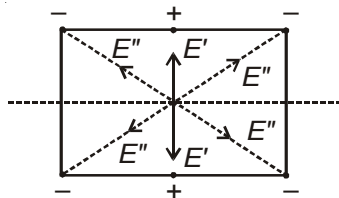
E_{net} at M is towards left i.e., $E \neq 0$ but $V = 0$

On rotating net current is zero, hence $B = 0, \mu = 0$



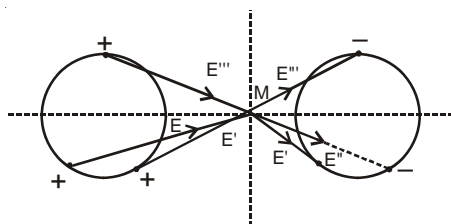
$E_{\text{net}} = 0$ (Same as p, angle between the electric fields is 120°) $V \neq 0$.

On rotating net current is not zero, hence $B \neq 0, \mu \neq 0$



$E_{\text{net}} = 0, V \neq 0$ On rotating net current is not zero hence

$B \neq 0, \mu \neq 0$



E_{net} is towards right i.e. $E \neq 0$ but $V = 0$ On rotating net current is zero hence $B = 0, \mu = 0$

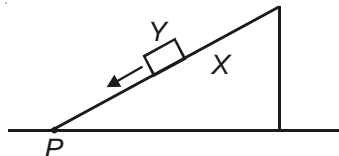
60. **Column II** shows five systems in which two objects are labelled as X and Y . Also in each case a point P is shown. **Column I** gives some statements about X and/or Y . Match these statements to the appropriate system(s) from **Column II**.

Column I

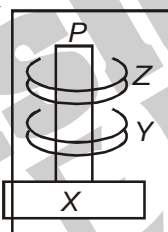
- (A) The forces exerted by X on Y has a magnitude Mg
- (B) The gravitational potential energy of X is continuously increasing

Column II

- (p) Block Y of mass M left on a fixed inclined plane X , slides on it with a constant velocity

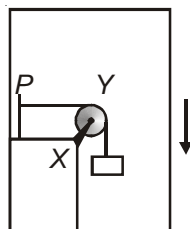


- (q) Two rings magnets Y and Z , each of mass M , are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity



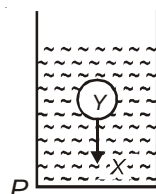
- (C) Mechanical energy of the system $X + Y$ is continuously decreasing

- (r) A pulley Y of mass m_0 is fixed to a table through a clamp X . A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system down is kept in a lift that is going with a constant velocity

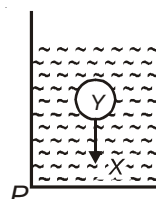


- (D) The torque of the weight of Y about point P is zero

- (s) A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid



- (t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container



Answer A(p,t), B(q, s, t), C(p, r, t), D(q)

Hints:

- (p) As block Y moves with constant velocity hence force exerted by X on Y will balance its weight Mg .
Work is done against friction force hence mechanical energy of the system decreases.
- (q) As lift is moving upwards hence gravitational potential energy of X is increasing.
Line of action of weight of Y passes through P hence torque is zero.
- (r) Gravitational potential energy is decreasing but kinetic energy is constant hence mechanical energy is decreasing.
- (s) As Y move down centre of gravity of X rises hence gravitational potential energy of X increases.
- (t) As Y moves with constant velocity hence force exerted by the X (liquid) will balance weight (Mg) of Y. Gravitational potential energy of X increases. Mechanical energy of system decreases as work is done against viscous force.

□ □ □

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